Information Acquisition, Signaling and Learning in Duopoly

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Abstract

We study firms' incentives to acquire private information in a setting where subsequent competition leads to firms' later signaling their private information to rivals. Due to signaling, equilibrium prices are distorted, and so while firms benefit from obtaining more precise private information, the value of information is reduced by the price distortion. Thus, compared with firms that do not attempt to manipulate rivals' beliefs, signaling firms acquire less precise information.

An industry-wide trade-association acquiring information increases firm profit and may also increase consumer surplus, so allowing such collective action may be in the interest of regulatory authorities.

Keywords: information acquisition, signaling, Bertrand competition, product differentiation

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1 Introduction

Firms' actions often reflect their private information. In imperfectly competitive dynamic settings, rival firms frequently extract information that is relevant for future competition from each other's price or output decisions, and use this information as a basis to determine future strategies. An implication of this is that in an attempt to soften future competition, firms may have an incentive to deliberately distort their actions in recognition of their rivals' drawing inferences about their underlying private information.

The canonical models of such signaling incentives postulate that firms are exogenously endowed with some private information on either costs or demand. For instance, Mailath (1989) and Bonatti et al. (forthcoming) assume that firms are endowed with private information on costs, whereas Caminal (1990), Gal-Or (1987), Jin (1994), and Mailath (1993) assume firms are endowed with private information on demand. In practice, however, rather than being endogenously endowed with such private information, firms often must make conscious and costly decisions to acquire such information, and the quality of this information hinges on the amount of resources committed to this acquisition by firms. For example, during the development of new products, firms often face cost or demand uncertainties and, thus, base their prices on forecasts.[1] Firms who hire managers with rich experience in operation or use the most up-to-date techniques to track and forecast costs or expend resources on demand estimates are more likely to make an accurate estimate of their actual costs or demand.

While the incentive to distort prices due to signaling has been extensively studied and is well understood, it is unclear how signaling incentives affect firms' initial information acquisition decisions. In this paper, we allow firms' private information to be chosen endogenously and focus on their information acquisition incentives in signaling games. Specifically, we address the following questions: How much information do firms acquire when their prices signal information to rivals? Does signaling increase or decrease firms' value of information? How do firms' information acquisition decisions differ from those of an industry-wide trade association or from those of a social planner?

In order to address these questions, we consider two firms that produce differentiated products

[1] See, e.g., Duane et al. (2010) or Jansen (2008).

and compete in prices in two periods, when their idiosyncratic costs are constant across time and are initially unknown to both firms.[2] Prior to the first-period market competition, firms simultaneously invest in information acquisition which results in (noisy) private signals about their costs. Thereafter, in the first period, firms simultaneously choose prices that are public information, but profits remain private information. Going into the second period, firms learn their own costs and, having observed the rival's price, firms glean information about the rival's costs. In the second period, firms compete in prices again and their second-period profits are realized.

When a firm invests in the accuracy of its initial cost signal, it faces a tradeoff between a gain in the first period and a loss in the second period. The first-period gain is immediate: More accurate forecasts on costs help firms better attune their first-period prices to actual costs; whereas more accurate initial cost estimates do not improve cost information in the second period, because by then firms already know their own costs through their first-period experiences. However, increased accuracy of initial cost-estimates increases the correlation between firms' second-period prices, and this—we show—reduces firms' second-period profits.

To see why the the accuracy of firms' private signals increases the correlation between second-period prices, consider the case in which goods are substitutes. Prices are strategic complements and so higher prices beget higher prices: The more accurate a firm's initial cost estimates are, the stronger is the positive correlation between its prices across time. Hence, its rival is more confident that the firm will charge a high second-period price after observing its high first-period price, and thus has a stronger incentive to also charge a high price in the second period. This price correlation in the second period is harmful to the firms due to imperfect pass-through of costs to prices: when a firm has a high cost, it charges a high price but has a low profit margin, whereas when it has a low cost, the firm charges a low price but has a high profit margin. Positive price correlation means that the rival firm will adjust its second-period price in a way that increases the firm's residual demand at low profit margins and decreases its residual demand at high profit margins, reducing the firm's profits on average.[3]

Despite the opposing effects, firms benefit from information acquisition, and the degree of

[2]In Section 6 we consider quantity competition and also demand uncertainty, where we show that the main findings carry over to both of these settings.

[3]The case of complements is analogous and follows readily once it is recognized that prices are strategic substitutes, so more accurate information increases the (negative) correlation of second-period prices.

information acquisition is uniquely determined by balancing the gain in total profit against the cost of information acquisition. The more independent the two goods are (i.e., weaker substitutes or weaker complements) the more firms invest in information acquisition as the negative second-period effects tied to price correlation are diminished.

To answer how firms' information acquisition decisions are affected by their signaling incentives, recall the nature of price distortions due to signaling. It is in a firm's interest to have its rival believe that it has a high cost as this will increase the demand for the firm: In the case of substitutes, the rival will price high if he believes that the firm has a high cost, which increases demand for the firm's product; and in the case of complementary goods, the rival will price low if he believes that the firm has a high cost, which also increases demand for the firm's product. Thus, a marginal increase in the first-period price above the level that maximizes the expected first-period profit induces (ceteris paribus) a first order gain in its expected second-period profit but only a second order loss in its expected first-period profit, and so firms price above the static optimal price.

At first glance, one may think that signaling incentives will induce firms to invest more in the quality of their information. The intuition rests on the reasoning that when a firm's cost forecast is more accurate, an increase in the firm's first-period price will convey a stronger message about its high cost and thus makes it easier for the firm to induce a high price from the rival in the next period. It turns out that this reasoning is not correct. Indeed, signaling always reduces firms' value of information. To see why, note that for a given level of investment in information acquisition, firms' expected second-period profits are the same regardless of whether first-period prices are distorted by signaling or not, because in equilibrium firms correctly infer their rival's private information. This implies that any difference between signaling and non-signaling firms' information acquisition decisions is driven by their consideration for the first-period profits.[4]

Both signaling and non-signaling firms gain in the first period when the precisions of their signals are improved because they can better attune their prices to their costs. However, non-signaling firms benefit more than signaling firms. To see why, note that a more precise signal will lead to

[4] The notion of non-signalling firms can be tied to the literature of information sharing. When firms commit to sharing their private information, the incentives to signal are eliminated. See, e.g., Li (1985), Vives (1984), Gal-Or (1985), Gal-Or (1986). Raith (1996) presents a synthesis of these models and Vives (2001) a comprehensive overview of the results. See also Jansen (2008) and Ganuza and Jansen (2013) who consider information disclosure after an acquisition decision is made. A critical distinction to our work, however, is that we maintain asymmetric information in the first period; allowing for information exchange only after the first period, but before the second period commences.

stronger updating away from prior beliefs and thus increase the variance of the firm's posterior cost-estimate. And because the firm's first-period price is linear in its conditional expectation of cost, the firm's first-period price will also have a larger variance. This variation in price dampens the value of information accrued to signaling firms compared to non-signaling firms, as the latter experience the variation around the ex ante optimal price and the Envelope Theorems applies, whereas for the latter prices are distorted by signalling and so the variation is more costly given that the profit function is concave in price.

Concerning welfare implications of firms' information acquisition decisions, we find that from a trade association's preperceptive the qualities of firms' information are inefficiently low. This is because firms fail to internalize positive externalities of their information on their rivals' second-period profits. Firms also acquire too little information when compared to the socially desirable level of information acquisition. Indeed, unless the goods are closely related (either strong substitutes or strong complements), even the trade association's preferred level of investment in information acquisition is too low compared to the social optimum—showing that industry-wide coordinated information acquisition may be pro-competitive.

There is a related literature on information acquisition in oligopolistic competition, but most papers only involve a single period interaction and consider how information acquisition is affected by the source of uncertainty and the nature of competition.[5] More recent papers, for example Angeletos and Pavan (2007), Colombo et al. (2014), Bernhardt and Taub (2015), Myatt and Wallace (2015), Myatt and Wallace (2016), focus on the efficiency of information acquisition and the usage of information acquired in an environment with both public and private signals. In contrast, our paper examines information acquisition from a different perspective: We study information acquisition in dynamic competition and focus on how signaling incentives affect firms' willingness to invest in information acquisition.

The rest of the paper is organized as follows: Sections 2 and 3 contain the model and the equilibrium. In Section 4 we consider welfare and derive comparisons to a trade association's incentives and those of a social planer, and in Section 5 we compare the equilibrium to the case in

[5]See, e.g., Li et al. (1987), Hwang (1993), Hwang (1995), Raju and Roy (2000), Sasaki (2001), Christen (2005). There is some work that considers dynamic models, e.g. Fudenberg and Tirole (1986), Riordan (1985) and Mirman et al. (1994), but there the information itself is generated through the market activity, rather than firms acquiring private information up-front.

which there are no signaling incentives. Some extensions are considered in Section 6 and there are concluding remarks in Section 7. Missing derivations and proofs are collected in the appendix.

2 The Model

Two risk neutral firms, i and j, produce differentiated products and compete in prices in two periods. Firms' constant marginal costs of production, denoted by c_i and c_j, are fixed across time, but their values are initially unknown. Specifically, c_i and c_j are i.i.d. random variables distributed according to $F(\cdot)$, with $E(c_i) = E(c_j) = \mu_c$ and $\text{Var}(c_i) = \text{Var}(c_j) = \sigma_c^2$ denoting the mean and variance.[6] Define $\tau_c := \frac{1}{\sigma_c^2}$ as the precision of the distribution of a firm's costs.

Prior to the first-period price competition, each firm can make a costly investment to acquire a private signal about its own cost. Let $s_i \in S_i$ denote Firm i's signal. Given c_i, signal s_i is drawn from the distribution $G(s_i|c_i)$. The variance of Firm i's signal given c_i is $\text{Var}(s_i|c_i) = \sigma_i^2$, and $\tau_i := \frac{1}{\sigma_i^2}$ denotes the precision of the signal s_i.

Firm i can choose the precision of its signal τ_i at a cost $k(\tau_i)$, where $k(\cdot)$ is a strictly increasing and convex C^2 function on $\mathbb{R}_+$ with $k(0) = 0$, $\lim_{\tau_i \to 0} k'(\tau_i) = 0$ and $\lim_{\tau_i \to \infty} k'(\tau_i) = \infty$. The objective of a firm is to maximize the sum of its expected profits from the two periods net of the costs of information acquisition. For simplicity of exposition, we assume that there is no discounting.

The representative consumer has a quadratic utility function which takes the form[7]

$$u\left(q_i, q_j, m\right) = \eta_0 \left(q_i + q_j\right) - \frac{1}{2} \left(\eta_1 q_i^2 + 2\eta_2 q_i q_j + \eta_1 q_j^2\right) + m, \tag{1}$$

where m is wealth, q_i, q_j are quantities consumed from the two firms, and η_0, η_1, η_2 are constants with $\eta_0 > 0$, $\eta_1 > |\eta_2| \geq 0$. The two goods are substitutes, independent or complements depending on whether $\eta_2 > 0$, $\eta_2 = 0$, or $\eta_2 < 0$. The two goods are perfect substitutes when $\eta_1 = \eta_2$ and perfect complements when $\eta_1 = -\eta_2$. The coefficient $\frac{\eta_2}{\eta_1} \in (-1, 1)$ is therefore a measure of the degree of product differentiation.

[6] All common cost components are assumed to be known and are normalized to zero and we assume that firms face uncertainty on their idiosyncratic costs. The case with demand uncertainty is discussed in Section 6.

[7] The quadratic utility function is commonly used in the literature to generate linear demand functions; see, for example, Singh and Vives (1984), Caminal (1990), Caminal and Vives (1996).

Given the two firms' prices p_{it}, p_{jt} in period $t = 1, 2$, the representative consumer chooses q_{it} and q_{jt} to maximizes her utility, which results in the following linear demand functions in period t:

$$q_{it} = a - bp_{it} + ep_{jt} \tag{2}$$

$$q_{jt} = a - bp_{jt} + ep_{it}; \tag{3}$$

where $a = \frac{\eta_0}{\eta_1+\eta_2}$, $b = \frac{\eta_1}{\eta_1^2-\eta_2^2}$ and $e = \frac{\eta_2}{\eta_1^2-\eta_2^2}$.

The timing of the game is illustrated in Figure 1 and is explained below:

Period 0 Information Acquisition: Nature draws c_i and c_j independently according to $F(\cdot)$ which is common knowledge. The realizations of c_i and c_j are unknown to both firms. Firms independently choose the precision of the signals, τ_i and τ_j. These choices are publicly observed, but the signals obtained are private information.[8]

Period 1 Price Competition: Given the firms' private signals about their own costs, s_i and s_j, the firms simultaneously choose publicly observed prices p_{i1} and p_{j1}. After the first-period production, markets clear and firms learn their realized costs which remain their private information.

Period 2 Price Competition: Firms engage in the second-period price competition by choosing p_{i2} and p_{j2} simultaneously.

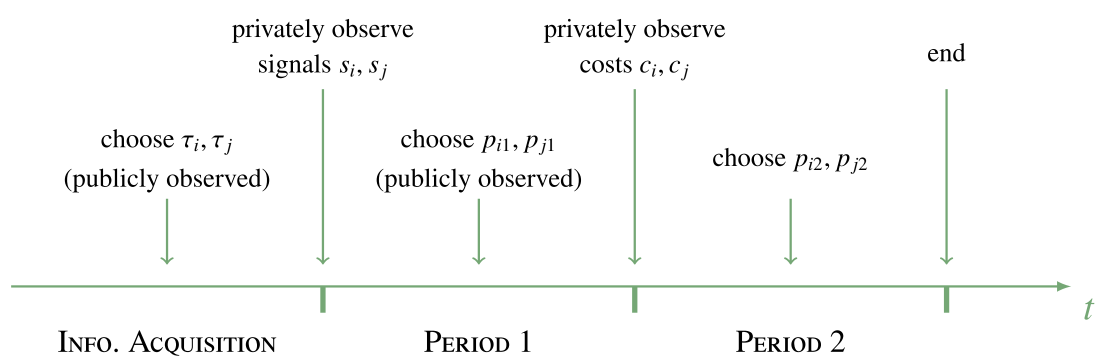

Figure 1: Timeline

The following assumption holds for the signal structure.

Assumption 2.1 *The information structure satisfies the following conditions:*

[8]The assumption of observable choices simplifies the derivation and exposition of the results. In Section 6 we show that our results continue to hold when τ_i, τ_j are unobservable. In particular, we show that a firm's equilibirum choice of precision is unaffected by whether the choice is observed or not.

- ***Unbiasedness:*** *$E(s_i|c_i) = c_i$ and $E(s_j|c_j) = c_j$.*

- ***Conditional Independence:*** *Conditional on c_i and c_j, s_i and s_j are independently distributed according to $G(s_i|c_i)$ and $G(s_j|c_j)$.*

- ***Affine Posterior Expectation:*** *$E(c_i|s_i)$ and $E(c_j|s_j)$ are affine in s_i and s_j.*

Several prior-posterior distribution functions give rise to an affine posterior expectation. For example, when $F(c_i)$ is a gamma distribution and $G(s_i|c_i)$ is a Poisson distribution with unknown mean c_i; when $F(c_i)$ is a beta distribution and $G(s_i|c_i)$ is a binomial distribution or when $F(c_i)$ and $G(s_i|c_i)$ are both normal.[9]

Given Assumption 2.1, Firm i's expected cost conditional on s_i is:[10]

$$E(c_i|s_i) = \overline{\tau}_i s_i + (1 - \overline{\tau}_i)\,\mu_c, \quad \text{where } \overline{\tau}_i := \frac{\tau_i}{\tau_i + \tau_c}. \tag{4}$$

Thus, a firm's posterior expectation of its cost upon observing signal s_i is a convex combination of the signal and the prior expectation on the cost, μ_c. When Firm i's signal is more precise, the posterior expectation puts a higher weight on the signal as compared to the prior mean. As a result, a more precise signal results in a larger dispersion (higher variance) of the conditional expectation $E(c_i|s_i)$.

As learning about and conveying information about costs is central to the analysis, in what follows we reserve the use of the expectations operator E to denote expected costs: $E(c|\cdot)$; whereas all other expectations operators are denoted by $\mathbb{E}$, with subscripts denoting the variable with respect to which the expectation is taken.

Strategies and Equilibrium Concept: Firm i's strategy is a triplet $(\tau_i, P_{i1}(\cdot), P_{i2}(\cdot))$; where $P_{i1}(\cdot)$ and $P_{i2}(\cdot)$ denote pricing functions (whereas p_{i1} and p_{i2} are the chosen prices). Define $\tau := (\tau_i, \tau_j)$, and $p_1 := (p_{i1}, p_{j1})$. For a given p_{j1}, Firm i infers that Firm j's private signal belongs to the set $P_{j1}^{-1}(p_{j1}) := \left\{s_j \middle| P_{j1}(s_j, \tau) = p_{j1}\right\}$. If Firm j's first-period pricing function is strictly monotone, the set $P_{j1}^{-1}(p_{j1})$ is a singleton and Firm i will perfectly infer s_j.

Given Firm j's strategy $\left(\tau_j, P_{j1}(\cdot), P_{j2}(\cdot)\right)$, Firm i's strategy is a best response if

[9] Affine information structures are common in the literature, see for example Li (1985), Gal-Or (1987), Gal-Or (1988), Chang and Lee (1992).

[10] See Ericson (1969).

1. $P_{i2}(c_i, p_1, \tau) \in \underset{p_{i2}}{\operatorname{argmax}} \mathbb{E}_{c_j}\left[(p_{i2} - c_i)q_{i2}(p_{i2}, P_{j2}(c_j, p_1, \tau)) \,\middle|\, s_j \in P_{j1}^{-1}(p_{j1}), \tau\right]$, for $p_{j1} \in P_{j1}(S_j, \tau)$ and $p_{i1} = P_{i1}(s_i, \tau)$. For a given (c_i, p_1, τ), the maximized value of Firm i's second-period profit is denoted by $\pi_{i2}(c_i, p_1, \tau)$.

2. $P_{i1}(s_i, \tau) \in \underset{p_{i1}}{\operatorname{argmax}} \mathbb{E}_{s_j}\mathbb{E}_{c_i}\left[(p_{i1} - c_i)q_{i1}(p_{i1}, P_{j1}(s_j, \tau)) + \pi_{i2}(c_i, p_{i1}, P_{j1}(s_j, \tau), \tau) \,\middle|\, s_i, \tau\right]$. Let $\Pi_i(s_i, \tau)$ denote the maximized value of firm i's expected profits from the two periods conditional on s_i and τ.

3. $\tau_i \in \underset{\tau_i}{\operatorname{argmax}} \mathbb{E}_{s_i}\left[\Pi_i\left(s_i, \tau_i, \tau_j\right)\right]$.

The strategy profile $(\sigma_i, \sigma_j) := \left\{\left(\tau_i, P_{i1}(\cdot), P_{i2}(\cdot)\right), \left(\tau_j, P_{j1}(\cdot), P_{j2}(\cdot)\right)\right\}$ is a Perfect Bayesian Equilibrium if

1. σ_i is a best response to σ_j and *vice versa*, and

2. for any subsets $\theta_i \subset S_i$ and $\theta_j \subset S_j$, $P_{i1}^{-1}(p_{i1}) = \theta_i$ if $p_{i1} = P_{i1}(\theta_i)$ and $P_{j1}^{-1}(p_{j1}) = \theta_j$ if $p_{j1} = P_{j1}(\theta_j)$.

There are two types of equilibrium configurations. The first type consists of separating configurations in which each firm's first-period pricing function is a one-to-one mapping. Hence, $P_{i1}^{-1}(p_{i1})$ and $P_{j1}^{-1}(p_{j1})$ contain a singleton $\forall p_{i1}, p_{j1}$. The second type consists of (semi-) pooling configurations in which a firm charges the same first-period price for different realizations of its signal. These latter configurations entail the firm not always differentiating its action on the basis of the information it obtains. Indeed, in the case of complete pooling, the firm makes no use of information at all; and as information is costly to obtain firms do not engage in any information acquisition.

Our interest is in how firms acquire and make use of information and so we focus on separating configurations in which the incentives to acquire and use information are the strongest. As we then show in Section 4, firms acquire too little information compared with a social planner or a trade association—a message that is reinforced in pooling or semi-pooling equilibria in which information is of less value to the firms.

3 Equilibrium

Given that we are deriving a separating equilibrium, firms' first-period private (noisy) information about their costs is fully revealed by their first-period prices. Hence, upon observing Firm j's first-period price, Firm i infers that Firm j's private signal is $\hat{s}_j := P_{j1}^{-1}(p_{j1})$, and *vice versa*. Here we use the hat above the signal to denote that this is the inference about the rival's signal, rather than the actual signal. (Of course, consistent beliefs imply that, in equilibrium, the inference is correct, i.e., $\hat{s}_j = s_j$.)

To appreciate why a firm's belief about the rival's signal matters consider the following: Upon observing its first-period profit, Firm j learns its (time-invariant) cost, which is used to determine its second-period price. Since profits are private information, Firm i does not learn Firm $j's$ cost—and therefore cannot fully anticipate Firm $j's$ second-period price. Nevertheless, since Firm j's first-period price is based on its private signal which contains information about its underlying cost, Firm i forms beliefs about Firm j's signal in order to glean information about Firm j's cost and subsequent pricing strategy. In short: Firm i infers Firm j's signal from j's first-period price, and uses this inference to update its belief about the distribution of Firm j's actual cost.

Of course, because firms use their rivals' first-period prices to shape posteriors which are then used to determine subsequent prices, firms have an incentive to distort their prices to skew beliefs and influence subsequent competition in the second period. We now show how learning and belief manipulation affect the pricing games and how this feeds back into the incentives of acquiring information in the first place.

We proceed using backward induction; starting with the second-period competition, given conjectures about rivals' costs; and then move back to characterize firms' first-period equilibrium pricing strategies. Finally, we consider firms' information acquisition decisions given the market competition continuation equilibrium. Throughout we restrict attention to interior solutions, postulating that the intercepts a are sufficiently high to always result in positive production.

For expositional convenience, we suppress firms' information acquisition decisions τ when denoting firms' pricing strategies $(P_{it}(\cdot), P_{jt}(\cdot))$ and profit functions $(\pi_{it}(\cdot), \pi_{jt}(\cdot))$. The notation τ is displayed explicitly only in the stage of information acquisition.

3.1 Second Period

Each firm learns its own cost after first-period production is completed, but the firms still remain uncertain about their rival's cost. Nevertheless, firms make inferences about their rivals' private signals $\hat{s}_i$ and $\hat{s}_j$ through their first-period prices, and use them to update the belief-distribution about their rivals' costs which are given by $F(c_i|\hat{s}_i)$ and $F(c_j|\hat{s}_j)$, respectively. Since $\hat{s}_j$ and $\hat{s}_i$ are based on observable first-period prices, they are common knowledge at the outset of the second period.

Firm i's problem in the second period is thus

$$\max_{p_{i2}} \int_{c_j} \Big(a - bp_{i2} + eP_{j2}(c_j)\Big)(p_{i2} - c_i)dF(c_j|\hat{s}_j); \tag{5}$$

and Firm j's problem is analogous.

Since firms' second-period expected profits are concave in their own prices, first order conditions are both necessary and sufficient, yielding:

$$p_{i2} = \frac{a}{2b} + \frac{c_i}{2} + \frac{e}{2b}\mathbb{E}_{c_j}\left[P_{j2}(c_j)\middle|\hat{s}_j\right], \tag{6}$$

$$p_{j2} = \frac{a}{2b} + \frac{c_j}{2} + \frac{e}{2b}\mathbb{E}_{c_i}\left[P_{i2}(c_i)\middle|\hat{s}_i\right]. \tag{7}$$

We have:

Lemma 3.1 *The second period game has a unique Bayesian Nash equilibrium. In the unique equilibrium, both firms adopt linear pricing strategies:*

$$P^*_{i2}(c_i;\hat{s}_i,\hat{s}_j) = \frac{a(2b+e)}{4b^2-e^2} + \frac{c_i}{2} + \frac{beE(c_j|\hat{s}_j)}{4b^2-e^2} + \frac{e^2E(c_i|\hat{s}_i)}{2(4b^2-e^2)}, \tag{8}$$

$$P^*_{j2}(c_j;\hat{s}_j,\hat{s}_i) = \frac{a(2b+e)}{4b^2-e^2} + \frac{c_j}{2} + \frac{beE(c_i|\hat{s}_i)}{4b^2-e^2} + \frac{e^2E(c_j|\hat{s}_j)}{2(4b^2-e^2)}. \tag{9}$$

Firms' equilibrium second-period prices depend on their own realized costs and each others' expectations on costs. Take Firm i's equilibrium price (8) as an example:

$$P^*_{i2}(c_i;\hat{s}_i,\hat{s}_j) = \frac{a(2b+e)}{4b^2-e^2} + \underbrace{\frac{c_i}{2}}_{\text{adaptation effect}} + \underbrace{\frac{beE(c_j|\hat{s}_j)}{4b^2-e^2} + \frac{e^2E(c_i|\hat{s}_i)}{2(4b^2-e^2)}}_{\text{strategic interaction effects}}.$$

The first term reflects how the demand intercept affects Firm $i's$ price. The second term shows by how much Firm i adapts its second-period price to its costs c_i. The final two terms capture the strategic interaction between firms' pricing strategies, which depends on both firms' posterior expectations on each other's costs.

When the goods are substitutes, then $e > 0$ and each of the final two terms is positive. Firms' prices are strategic complements. Specifically, the third term shows that Firm i raises p_{i2} when it expects an increase in Firm j's cost. This is because Firm i anticipates that Firm j raises p_{j2} due to Firm j's adaptation effect. Since firms' pricing strategies are strategic complements, Firm i increases p_{i2} as well. The last term shows that Firm i's price also increases in its rival's posterior expectation on Firm i's cost. Using the same argument for Firm i, Firm j raises p_{j2} in response to a more optimistic posterior expectation about Firm i's cost. As a consequence, Firm i also increases p_{i2} due to the strategic complementarity effect.

The case of complementary goods differs slightly. Since $e < 0$ for complementary goods, the third term is negative. Note, however, that the final term is positive also for the case of complements: if the rival expects Firm i's cost to be high, it anticipates a higher price from Firm i. Given strategic substitutes, the rival lowers its price accordingly, which—again due to strategic substitutability—leads firm Firm i to increase its price.

In moving forward, let $\pi^*_{i2}(c_i, \hat{s}_i, \hat{s}_j)$ denote Firm i's expected second-period equilibrium profit:

$$\pi^*_{i2}(c_i, \hat{s}_i, \hat{s}_j) := \pi^*_{i2}(c_i, P^*_{i2}, P^*_{j2}) = \left(a - bP^*_{i2}(c_i, \hat{s}_i, \hat{s}_j) + e\mathbb{E}_{c_j}\left[P^*_{j2}(c_j, \hat{s}_i, \hat{s}_j)\middle|\hat{s}_j\right]\right)(P^*_{i2}(c_i, \hat{s}_i, \hat{s}_j) - c_i), \tag{10}$$

with P^*_{i2} and P^*_{j2} defined in (8) and (9), respectively.

3.2 First period: signaling and belief manipulation

Consider first-period price competition. Firm i receives a private signal s_i about its own cost c_i and updates the distribution on c_i to $F(c_i|s_i)$. The information available to Firm i in this stage is s_i, τ_i, τ_j. Firm i expects its rival's first-period price p_{j1} to be a function of the rival's signal s_j which is unobservable to Firm i. Conditional on signal s_i, Firm i's expected first-period profit from charging p_{i1} is given by

$$\begin{aligned}\Pi_{i1}(p_{i1}, P_{j1}|s_i) &:= \int_{s_j}\int_{c_i}\Big(a - bp_{i1} + eP_{j1}(s_j)\Big)(p_{i1} - c_i)dF(c_i|s_i)dG_j(s_j) \\ &= \Big(a - bp_{i1} + e\mathbb{E}_{s_j}\left[P_{j1}(s_j)\right]\Big)(p_{i1} - E(c_i|s_i)). \end{aligned} \tag{11}$$

Conditional on s_i, Firm i's expected second-period profit from charging p_{i1} is

$$\Pi^*_{i2}(\hat{s}_i|s_i) := \int_{c_i}\int_{\hat{s}_j}\pi^*_{i2}(c_i, \hat{s}_i, \hat{s}_j)dG_j(\hat{s}_j)dF(c_i|s_i), \tag{12}$$

where $\hat{s}_i = P^{-1}_{i1}(p_{i1})$ and $\hat{s}_j = P^{-1}_{j1}(p_{j1})$.

Firm i's problem in the first period is to choose price p_{i1} to maximize the sum of profits from the two periods:

$$\max_{p_{i1}} \Pi_{i1}(p_{i1}, P_{j1}|s_i) + \Pi^*_{i2}(\hat{s}_i|s_i). \tag{13}$$

Using (11) and (12), the first order condition is

$$a + bE(c_i|s_i) - 2bp_{i1} + e\mathbb{E}_{s_j}\left[P_{j1}(s_j)\right] + \frac{\partial\Pi^*_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}} = 0. \tag{14}$$

Firm j's problem is analogous to Firm i's problem. By symmetry, we have

$$a + bE(c_j|s_j) - 2bp_{j1} + e\mathbb{E}_{s_i}\left[P_{i1}(s_i)\right] + \frac{\partial\Pi^*_{j2}(\hat{s}_j|s_j)}{\partial p_{j1}} = 0. \tag{15}$$

Firms' optimal first-period prices jointly solve (14) and (15). Note that when firms choose prices to maximize their first-period profits, the optimal static prices solve (14) and (15) after setting $\frac{\partial\Pi^*_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}} = \frac{\partial\Pi^*_{j2}(\hat{s}_j|s_j)}{\partial p_{j1}} = 0$. The solution is referred to as the optimal *non-signaling prices* because firms do not attempt to manipulate their rivals' beliefs through signaling.

As noted, $\frac{\partial\Pi^*_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}}$ captures Firm i's distortion of the first-period price from the optimal non-signaling price in order to manipulate the rival's belief about the market environment in the second period. Specifically,

$$
\begin{aligned}
\frac{\partial \Pi_{i2}^{*}(\hat{s}_i|s_i)}{\partial p_{i1}} &= \int_{c_i}\int_{\hat{s}_j} \frac{\partial \pi_{i2}^{*}(c_i, \hat{s}_i, \hat{s}_j)}{\partial p_{i1}} dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \int_{c_i}\int_{\hat{s}_j} \frac{\partial \pi_{i2}^{*}(c_i, P_{i2}^{*}, P_{j2}^{*})}{\partial P_{j2}^{*}} \frac{\partial P_{j2}^{*}}{\partial E(c_i|\hat{s}_i)} \frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}} dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \int_{c_i}\int_{\hat{s}_j} e(P_{i2}^{*} - c_i) \frac{be\bar{\tau}_i}{4b^2 - e^2} \frac{\partial \hat{s}_i}{\partial p_{i1}} dG_j(\hat{s}_j) dF(c_i|s_i), \qquad (16)
\end{aligned}
$$

where the second equality follows from (10) and the Envelope Theorem, and the third equality follows from (9) and (4).

Assuming for the moment that $P_{i1}(s_i)$ and $P_{j1}(s_j)$ are differentiable (which is confirmed in equilibrium), by the Inverse Function Theorem $\frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{1}{P'_{i1}(s_i)}$. Given $P_{i2}^{*} - c_i > 0$, $\frac{\partial \Pi_{i2}^{*}(\hat{s}_i|s_i)}{\partial p_{i1}} > 0$ if $P'_{i1}(s_i) > 0$. So, when firms' first-period prices are strictly increasing in their private signals, firms gain from distorting their first-period prices above the optimal non-signaling prices. To see this, note that when a firm raises its first period price, by the Envelope Theorem, it will affect the firm's expected second-period profit through its rival's second-period price. Specifically, by raising p_{i1} marginally, Firm i shifts up the rival's posterior expectation of Firm i's marginal cost by

$$
\frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}} = \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{\bar{\tau}_i}{P'_{i1}(\hat{s}_i)}. \qquad (17)
$$

As a consequence, Firm j will raise its second-period price by $\frac{be}{4b^2-e^2}\frac{\bar{\tau}_i}{P'_{i1}(s_i)}$ when the goods are substitutes and reduce its price by the same magnitude when they are complements. In both cases, second-period competition is relaxed which benefits both firms (refer to (9)). Thus, Firm i has a first-order gain in its second-period profit and a second-order loss in its first-period profit by distorting its first-period price above the optimal non-signaling price.

Next, we consider the equilibrium of the entire two-period pricing game by considering the first-period optimal strategies for given updated beliefs on firms' own costs. The quadratic utility function (i.e., linear demand) together with linear posterior expectations result in a unique fully revealing equilibrium:

Proposition 1 *There is a unique fully revealing equilibrium in the continuation game starting in* *the first period. In the equilibrium, firms' first-period pricing functions are given by*

$$P_{i1}^*(s_i) = \alpha_1^* E(c_i|s_i) + \beta_1^*, \tag{18}$$

$$P_{j1}^*(s_j) = \alpha_1^* E(c_j|s_j) + \beta_1^*, \tag{19}$$

with

$$\alpha_1^* = \frac{2b^2 - e^2}{4b^2 - e^2}, \text{ and} \tag{20}$$

$$\beta_1^* = \frac{e\mu_c(4b^4 - 3b^2e^2 + e^4) - 4ab^2e^2 + abe^3 + 8ab^4 + ae^4}{(4b^2 - e^2)(2b^2 - e^2)(2b - e)}. \tag{21}$$

To illustrate how signaling incentives distort firms' first-period prices, we derive the optimal non-signaling first-period price explicitly. Setting $\frac{\partial \Pi_{i2}^*(\hat{s}_i|s_i)}{\partial p_{i1}} = \frac{\partial \Pi_{j2}^*(\hat{s}_j|s_j)}{\partial p_{j1}} \equiv 0$ in (14) and (15) and solving for p_{i1} and p_{j1}, it can be verified that the optimal non-signaling first-period prices are:

$$p_{i1}^{NS} = \frac{1}{2}E(c_i|s_i) + \frac{2a + e\mu_c}{2(2b - e)}, \tag{22}$$

$$p_{j1}^{NS} = \frac{1}{2}E(c_j|s_j) + \frac{2a + e\mu_c}{2(2b - e)}. \tag{23}$$

Since $|e| < b$, the optimal non-signaling first-period pricing function is steeper than the signaling first-period pricing function as is illustrated in Figure 2. The blue line (P_{i1}^*) represents the first-period equilibrium pricing function (18) when firms signal their private information through prices. By contrast, the red line (P_{i1}^{NS}) represents the non-signaling first-period pricing function. Once the the red line crosses the 45-degree line Firm i stops producing. The blue line is uniformly higher than the red line which results from firms' signaling incentives. The divergence between the two pricing functions decreases in Firm $i's$ expected cost. This implies that firms' signaling incentives weaken when their expected costs increase. Intuitively, firms' prices are bounded above by the demand intercept. When firms' expected costs go up, the range for feasible prices (between expected cost and the demand intercept) becomes narrower and hence there is less room for firms to distort their price upward.

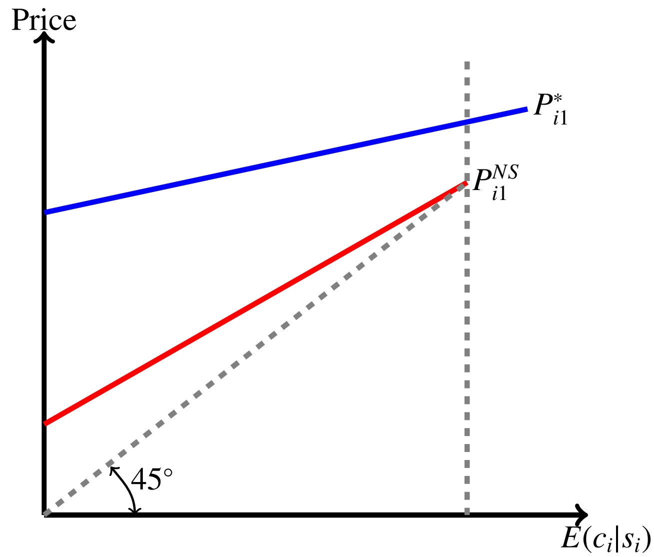

Figure 2: Comparison of First Period Pricing Functions

3.3 Information Acquisition

Before engaging in price competition in the first period, firms glean information about their costs by investing in the generation of an unbiased signal s. When a firm chooses the quality of its signal, it internalizes the impact of its improved private information on its profits in both periods. We analyze how the signal precision affects a firm's first- and second-period expected profits separately. To begin, we start with the firm's expected first-period profit. Define $\Pi_i^*(s_i) := \int_{s_j} \pi_i\left(P_{i1}^*(s_i), P_{j1}^*(s_j)\right) dG_j(s_j)$.

Lemma 3.2 *Firm i's expected first-period equilibrium profit is:*

$$\mathbb{E}_{s_i}\Pi_{i1}^*(s_i) = \frac{2b^3(2b^2 - e^2)}{(4b^2 - e^2)^2}\frac{\tau_i}{(\tau_c + \tau_i)\tau_c} + \Psi; \tag{24}$$

where Ψ is a collection of terms not involving τ_i or τ_j.

1. *Firm i's expected first-period profit increases in the precision of its signal. The marginal gain is*

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^*(s_i)}{\partial \tau_i} = \frac{2b^3(2b^2 - e^2)}{(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_i)^2} > 0. \tag{25}$$

2. *Firm i's expected first-period profit is independent of the quality of Firm j's private informa-*

tion. That is,

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^*(s_i)}{\partial \tau_j} = 0.$$

In the first period, each firm's price is only based on the conditional expectation of its own cost. Since firms' costs are idiosyncratic, their first-period prices are independent. As a result, when a firm increases its investment in information acquisition, the firm can better adjust its first-period price to its actual cost, but this does not affect its rival's first-period price. The firm therefore has a gain in its expected first-period profit.

To see that the firm's expected first-period profit is independent of its rival's investment in information acquisition, note that the firm's profit is linear in its rival's price. When Firm j increases the precision of s_j, it increases the variance of its conditional expectation $E(c_j|s_j)$, which in turn increases the variance of Firm j's first-period price. However, it leaves unaffected Firm i's expectation of Firm j's price, and as p_{j1} enters Firm i's profit linearly, any gains and losses from variations in p_{j1} offset each other. As a result, Firm i's expected first-period profit is independent of Firm j's investment in information acquisition.

Consider now the effect of signal precision on second-period expected profits, which is considerably more nuanced. Note first that upon observing its first period profit, the firm knows its costs with certainty, regardless of the initial choice of precision of the signal it obtained. Thus, there is no incentive to increase the precision of the signal in order to have better information on one's own costs in the second period.

However, whereas firms' first-period prices are independent, their second-period prices are correlated through two channels, and the price correlation increases in firms' investment in information acquisition. Recall from (8) and (9) that each firm's second-period price depends on three things: its own realized cost, the rival's posterior expectation of its realized cost, and its posterior expectation of the rival's realized cost. The first channel through which second-period prices are correlated is that firms' realized costs are positively correlated with their rivals' conjectures of their realized costs. For example, c_i in (8) is positively correlated with $E(c_i|\hat{s}_i \equiv s_i)$ in (9). And the second source of correlation is that firms' posterior expectations of rivals' costs enter both firms' prices.

While a more precise signal s_i does not affect the firm's knowledge of its own costs c_i, it will increase the correlation between Firm i's realized cost c_i and Firm j's conjecture of Firm i's cost $E(c_i|\hat{s}_i \equiv s_i)$, which in turn, increases the price correlation. The price correlation is reinforced through the second channel. To see this, when Firm i increases the precision of its signal, it increases the variance of $E(c_i|s_i)$ which enters both firms' pricing functions and results in a larger co-movement between firms' second-period prices. In short, firms' second-period prices will be more positively correlated for substitutes and more negatively correlated for complements.

Regardless of whether the goods are substitutes or complements, the effect of an increase in the correlation of prices reduces firms' second-period expected profits. We use substitutes as an example to illustrate the intuition and the case of complements follows the same logic. When a firm has a high cost, due to incomplete pass-through of costs to prices, it will charge a high price but experience a reduction in the profit margin. Since prices are positively correlated, the rival's price is now more likely to also be higher. While this is good news, the good news comes only when the profit-margin is low due to high costs. Now consider a low cost. Due to the increased profit-margin associated with lower costs, the firm optimally lowers its price to increase sales. However, when prices are positively correlated, the rival's price is also more likely to be low and so the firm is unable to take full advantage of its increased profit margin as the rival steals some demand. In sum, when there is co-movement of prices, a rival's price tend to increase a firm's demand when margins are low, and reduces a firm's demand when margins are high.

Thus, when firms' signals become more precise, their second-period prices are more correlated and this lowers average second-period profits. This finding may be viewed as a manifestation of Gal-Or (1986), who finds that keeping information private is a dominant strategy in Bertrand competition with private costs. In our model, acquiring more precise information is equivalent to sharing more information with the rival in the second-period competition, and hence is harmful for a firm's expected second-period profit.

We formalize the above discussion in the following lemma:

Lemma 3.3 *Firm i's expected second-period profit is*

$$\mathbb{E}_{s_i}\Pi^*_{i2}(s_i) = -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau_i}{(\tau_c+\tau_i)\tau_c} + \frac{b^3e^2}{(4b^2-e^2)^2}\frac{\tau_j}{(\tau_c+\tau_j)\tau_c} + \Psi, \tag{26}$$

where the term Ψ does not involve τ_i or τ_j.

1. *Firm i's expected second-period profit decreases in the precision of its signal. The marginal loss is*

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^*(s_i)}{\partial \tau_i} = -\frac{be^2(8b^2 - 3e^2)}{4(4b^2 - e^2)^2} \frac{1}{(\tau_i + \tau_c)^2} < 0. \tag{27}$$

2. *Firm i's expected second-period profit increases in the precision of Firm j's signal and its marginal gain is*

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^*(s_i)}{\partial \tau_j} = \frac{b^3 e^2}{(4b^2 - e^2)^2} \frac{1}{(\tau_j + \tau_c)^2} > 0. \tag{28}$$

An implication of Lemma 3.3 is that a firm's investment in information acquisition constitutes a positive externality on its rival's expected second-period profit. This is because when a firm's first-period price is more informative about its cost, it is easier for the rival to predict the firm's second-period price and to best response accordingly.

Firm i's objective in the information acquisition stage is to maximize the net expected profits from the two periods, namely

$$\max_{\tau_i} \mathbb{E}_{s_i} \left[\Pi_{i1}^*(s_i|\tau_i) + \Pi_{i2}^*(s_i|\tau_i) - k(\tau_i) \right] \tag{29}$$

Given the convexity of $k(\cdot)$, the objective function is strictly concave in τ_i. We have shown that Firm i gains in the first period but loses in the second period when its signal becomes more accurate. Following (25) and (27), it can be verified that Firm i's total gain from a more precise signal is

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^*(s_i|\tau_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^*(s_i|\tau_i)}{\partial \tau_i} = \frac{b(4b^2 - 3e^2)}{4(4b^2 - e^2)} \frac{1}{(\tau_i + \tau_c)^2} > 0. \tag{30}$$

Firm i chooses the precision τ_i to balance its marginal gain in total profit and the marginal cost of information acquisition. The solution is summarized in the following proposition:

Proposition 2 *In the unique fully revealing equilibrium, firms make the same equilibrium information acquisition investment τ^*, which is given by*

$$\frac{b(4b^2 - 3e^2)}{4(4b^2 - e^2)} \frac{1}{(\tau_c + \tau^*)^2} = k'(\tau^*). \tag{31}$$

Taking the derivative of the firms' marginal benefits from information acquisition (left hand side of (31)) with respect to the degree of substitution/complemetarity between the goods, it gives $\frac{-4b^3e}{(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau^*)^2}$. This derivative is negative for substitutes and positive for complements. So firms will acquire more information when goods are more independent because the negative impact of information acquisition on firms' second-period profit diminishes as the correlation between second-period prices becomes smaller.

4 Welfare Effects of Information Acquisition

In this section, we investigate how the firms' information acquisition decisions affect welfare. It is well understood in the literature that signaling incentives result in inefficiently high prices in the context of duopoly price competition, but little is known about its impact on the efficiency of information acquisition. To fill the gap, we focus on the latter and study how firms' information acquisition decisions differ from two natural benchmarks. First, we consider a trade association that operates as an information gathering agent on behalf of the firms, while assuming that firms remain non-cooperative in the downstream market. The trade association, thus, accounts for and internalizes any externalities associated with information gathering that exist between the two rivals. Second, we account for the impact of information acquisition on consumer surplus, by studying how a social planer's preferred levels of information acquisition differ from those of the trade association. For both benchmarks we assume that prices are still set non-cooperatively; that is, neither the trade association nor the social planner can affect the prices that the firms chose once they obtain cost information.

4.1 Trade Association

Consider a trade association that maximizes the firms' joint (non-cooperative) profits from the two periods (net of the cost of information acquisition), given the non-cooperative equilibrium of the pricing game. That is, the trade association chooses τ_i and τ_j to maximize firms' joint profit, while maintaining that firms' first-period and second-period prices are determined by ((18), (19)) and ((8), (9)). We let TP_t^* denote firms' joint (i.e., total) equilibrium profits in period t.

From the second part of Lemma 3.2, Firm j's expected first-period profit is independent of

Firm i's signal precision τ_i. Hence, a firm's information acquisition decision does not entail any externality on the rival's first-period profit. Thus,

$$\frac{\partial TP_1^*}{\partial \tau_i} = \frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^*(s_i)}{\partial \tau_i} + \cancelto{0}{\frac{\partial \mathbb{E}_{s_i} \Pi_{j1}^*(s_j)}{\partial \tau_i}} = \frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^*(s_i)}{\partial \tau_i}, \tag{32}$$

where $\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^*(s_i)}{\partial \tau_i}$ is given by (25).

In contrast, from the second part of Lemma 3.3, Firm j's expected second-period profit increases in the precision of Firm i's signal, so

$$\frac{\partial TP_2^*}{\partial \tau_i} = \frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^*(s_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_j} \Pi_{j2}^*(s_j)}{\partial \tau_i} > \frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^*(s_i)}{\partial \tau_i}, \tag{33}$$

with $\frac{\partial \mathbb{E}_{s_j} \Pi_{j2}^*(s_j)}{\partial \tau_i}$ given in (28).

Thus, the trade association must account for a positive externality of one firm's signal precision on the other firm's second period profit, when setting the optimal information acquisition decision.

Proposition 3 *In the non-cooperative pricing equilibrium, due to the positive externality of one firm's investment on the rival's second-period expected profits, firms acquire too little information from a joint profit maximization perspective. Given the trade association's mandate, the optimal choice of signal precision is larger than in the base model and is uniquely determined by*

$$\frac{b\left(\left(4b^2 - e^2\right)\left(4b^2 - 3e^2\right) + 4b^2 e^2\right)}{4(4b^2 - e^2)^2} \frac{1}{(\tau_i^{TA} + \tau_c)^2} = k'(\tau_i^{TA}), \quad i = 1, 2. \tag{34}$$

4.2 Social Planer

Coordinated actions among competitors designed to increase their profits are generally subject to antitrust and regulatory concerns, even when market competition remains non-cooperative and competitive. Thus, it is of interest to determine how the trade association's activities affect total welfare. To get at this question, consider a social planner whose objective is to choose τ_i and τ_j to maximize the aggregate social welfare, namely the sum of consumer surplus and industry profit across the two periods, net of information acquisition costs.

Social welfare in period t is the sum of consumer surplus and firms' profits:

$$\begin{aligned} SW_t &= CS_t + TP_t \\ &= u(q_{it} + q_{jt}) - c_i q_{it} - c_j q_{jt} \\ &= \eta_0(q_{it} + q_{jt}) - \frac{1}{2}\left(\eta_1 q_{it}^2 + 2\eta_2 q_{it} q_{jt} + \eta_1 q_{jt}^2\right) - c_i q_{it} - c_j q_{jt}. \end{aligned} \tag{35}$$

Taking expectation over quantities and costs, expected social welfare can be written as

$$\mathbb{E}\left[SW_t\right] = -\frac{1}{2}\eta_1\left(\mathrm{Var}(q_{it}) + \mathrm{Var}(q_{jt})\right) - \eta_2 \mathrm{Cov}(q_{it}, q_{jt}) - \mathrm{Cov}(c_i, q_{it}) - \mathrm{Cov}(c_j, q_{jt}) + \Psi, \tag{36}$$

where Ψ only contains terms that do not involve τ_i or τ_j. From this one obtains:

Proposition 4 *The social planner's optimal choice of investment in firms' information acquisition τ^S is uniquely determined by*

$$\frac{b\left(48b^4 - 44b^2e^2 + 9e^4\right)}{8(4b^2 - e^2)^2}\frac{1}{(\tau^S + \tau_c)^2} = k'(\tau^S), \quad i = 1, 2. \tag{37}$$

Compared to the socially optimal levels of information acquisition, the firms' independently chosen levels are too low. Moreover, even the trade association's desired levels of information acquisition are too low, unless the goods are closely related (i.e., $|e|$ is close to b).

The difference between the social planner and the trade association's payoff captures consumer surplus. Proposition 4 implies that the degree to which consumers benefit from firms' investments in information acquisition hinges on the degree of substitution/complementarity between the two goods. To understand the tradeoff, the utility function (1) and the inverse demand functions based on (2) and (3) are used to write consumer surplus in period t as:

$$CS_t = u(q_{it}, q_{jt}) - p_{it} q_{it} - p_{jt} q_{jt} = \frac{\eta_1}{2}\left(q_{it}^2 + q_{jt}^2\right) + \eta_2 q_{it} q_{jt}; \tag{38}$$

and the sum of the expected consumer surplus from the two periods is

$$\begin{aligned}\sum_{t=1}^{2}\mathbb{E}\left[CS_t\right] &= \frac{\eta_1}{2}\left(\sum_{t=1}^{2}\mathbb{E}\left[q_{it}^2\right]+\sum_{t=1}^{2}\mathbb{E}\left[q_{jt}^2\right]\right)+\eta_2\sum_{t=1}^{2}\mathbb{E}\left[q_{it}q_{jt}\right]\\ &= \frac{\eta_1}{2}\left(\sum_{t=1}^{2}\mathrm{Var}(q_{it})+\sum_{t=1}^{2}\mathrm{Var}(q_{jt})\right)+\eta_2\sum_{t=1}^{2}\mathrm{Cov}\left(q_{it},q_{jt}\right)+\Psi. \end{aligned} \tag{39}$$

This expression shows that consumers benefit from variation in quantities, but suffer from co-movement between quantities, as when $\eta_2 > 0$ (substitutes), we have $\mathrm{Cov}\left(q_{it}, q_{jt}\right) < 0$, whereas with complements we have $\eta_2 < 0$ and $\mathrm{Cov}\left(q_{it}, q_{jt}\right) > 0$.

We begin by investigating the impact of firms' investments on the variance in quantities. Recall that first-period prices are independent and linear in firms' conditional cost expectations. When a firm's information becomes more accurate, its first-period pricing strategy will increase the weight on the firm's private signal and reduce the weight on its *ex ante* expected cost, whereas the rival's first-period pricing strategy is not affected. This results in a larger variance in the firm's first-period price, which directly translates to a larger variance in its first-period quantity.

The second period is a bit more complex. Recall that firms' second-period prices are positively (negatively) correlated for substitutes (complements) and the degree of correlation increases in firms' investments. So, more investment in a firm's information acquisition will result in a smaller variance in the firm's second-period quantity. Take substitutes as an example. As a firm's signal becomes more accurate, when the firm adjusts its second-period price according to its realized cost, the rival is more likely to adjust price in the same direction, which will dampen the change in the firm's quantity. The second effect becomes more pronounced when the degree of substitution increases. As a result, when the goods are close to perfect substitutes, the second effect dominates, and an increase in the firm's investment reduces consumer welfare through its impact on the variance of each of the firms' quantities.

Next, we examine the impact of firms' investment on the covariance of their quantities. First, note that the covariance in quantities is negative (positive) for substitutes (complements) in the first period and positive (negative) for substitutes (complements) in the second period. To see this, again, take substitutes as an example. Since first-period prices are independent, an increase in a firm's price decreases its own quantity but increases its rival's quantity. The negative correlation between quantities is stronger when the degree of substitution increases. In the second period, positively correlated prices result in positively correlated quantities. When the firm increases its investment, it increases the co-movement between quantities in each period, which offsets each other. Nevertheless, when the degree of substitution is large, the first effect dominates and the firm's investment reduces consumer welfare through its impact on co-movement between quantities.

Despite the fact that the trade association acquires too little or too much information from the social planner's perspective—depending on how closely related the two firms' goods are—individual firms' investments are always inefficiently low. This is because individual firms fail to internalize the positive externality of their investment on the rival's profit. This positive externality dominates the negative externality of investment on consumer welfare when the degree of substitution/complementarity between the goods is large. An implication of this is that even in cases where the trade association acquires too much information from a social perspective, the trade association may still lead to better outcomes than when the trade associations' activities are prohibited.

5 Comparison with Non-Signaling Firms

In this section, we investigate how signaling incentives impact firms' information acquisition decisions, and how this affects welfare. To this end we use non-signaling firms' decisions of information acquisition as a benchmark and compare it to that of signaling firms. This comparison is not only of theoretical interest but also has policy implications. The existing literature (for instance Mailath (1989)) focuses on welfare consequences of price signaling when firms' private information is exogenously given. Our study investigates how price signaling affects the efficiency of firms' information acquisition decisions and provides a complete picture of the welfare consequences of policies eliminating firms' signaling incentives, such as allowing for the exchange of private information before the second period commences.

5.1 Information Acquisition

The comparison between signaling and non-signaling firms' information acquisition decisions are illustrated in Figure 3 and summarized in the following proposition:

Proposition 5 *Signaling firms acquire less information than non-signaling firms and the divergence between the amount of information acquired by signaling and non-signaling firms increases in the degree of substitution or complementarity between the goods.*

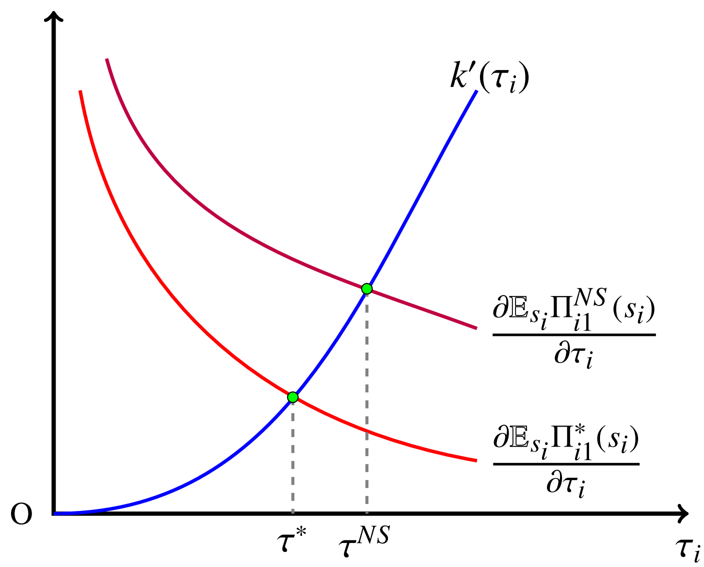

Figure 3: Comparison of τ^* and τ^{NS}

In Figure 3, the upward sloping curve indicates firms' marginal cost from information acquisition. The two downward sloping curves $\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)}{\partial \tau_i}$ and $\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{*}(s_i)}{\partial \tau_i}$ are non-signaling and signaling firms' marginal gains from information acquisition, respectively. The figure shows that signaling firms acquire less information than non-signaling firms because they derive a smaller gain.

Recall that in equilibrium any distortions in the first-period pricing decisions due to signaling are anticipated and so second-period beliefs and therefore the second-period pricing-decisions are not actually affected by signaling. Hence, the reason why signaling firms acquire less information than non-signaling firms is tied entirely to first-period profit considerations. Starting with non-signaling firms, for given signal precisions, Firm i's equilibrium first-period profit is

$$\Pi_{i1}^{NS}(s_i) = \left(a - bP_{i1}^{NS}(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^{NS}(s_j)\right]\right)\left(P_{i1}^{NS}(s_i) - E(c_i|s_i)\right), \tag{40}$$

where $P_{i1}^{NS}(s_i)$ and $P_{j1}^{NS}(s_j)$ are determined by (22) and (23), respectively. Note that since Π_{i1}^{NS} depends on the precision of Firm i's signal only through $E(c_i|s_i)$, we can treat Π_{i1}^{NS} as a function of $E(c_i|s_i)$. Hence, to evaluate Firm i's marginal gain from information acquisition in the first period, it suffices to investigate how Π_{i1}^{NS} changes in $E(c_i|s_i)$. Taking the derivative yeilds:

$$\frac{d\Pi_{i1}^{NS}(E(c_i|s_i))}{dE(c_i|s_i)} = \frac{\partial \Pi_{i1}^{NS}}{\partial E(c_i|s_i)} + \cancelto{0}{\frac{\partial \Pi_{i1}^{NS}}{\partial P_{i1}^{NS}} \frac{\partial P_{i1}^{NS}}{\partial E(c_i|s_i)}} \tag{41}$$

$$= -\underbrace{\left(a - bp_{i1}^{NS}(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^{NS}(s_j)\right]\right)}_{q_{i1}^{NS}}. \tag{42}$$

The impact of a change in $E(c_i|s_i)$ in Firm i's first-period expected profit can be decomposed into a direct effect (the first term in (41)) and an indirect effect through P_{i1}^{NS} (the second term in (41)). Since non-signaling firms choose first-period prices to maximize their first-period profit, by the Envelope Theorem, a change in $E(c_i|s_i)$ does not have an indirect effect on profits and the second term in (41) is 0. The expression in (42) shows that an increase in expected cost $E(c_i|s_i)$ reduces Firm i's profit by the number of units that are affected by the marginal increase in costs, i.e., the firm's output q_{i1}^{NS}.

Recall that when Firm i increases the precision of its signal, it also increases the variance of $E(c_i|s_i)$. The question is whether Firm i benefits *ex ante* from a larger variance in $E(c_i|s_i)$. If Firm i's first-period quantity q_{j1}^{NS} were constant in $E(c_i|s_i)$, its gain from a decrease in $E(c_i|s_i)$ would cancel out its loss from an increase in $E(c_i|s_i)$. However, as P_{i1}^{NS} is strictly increasing in $E(c_i|s_i)$, Firm i reduces the quantity q_{i1}^{NS} when $E(c_i|s_i)$ increases and increases the quantity when $E(c_i|s_i)$ decreases. This implies that the loses to Firm i when $E(c_i|s_i)$ increases are smaller than the gain to the firm when $E(c_i|s_i)$ decreases, and therefore the firm benefits from more accurate private information due to the positive direct impact.

We use the same approach to evaluate the impact of information acquisition on signaling firms' first-period profits and draw a comparison with no-signaling firms. Conditional on s_i, Firm i's first-period equilibrium profit is

$$\Pi_{i1}^{*}(E(c_i|s_i)) = \left(a - bP_{i1}^{*}(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^{*}(s_j)\right]\right)\left(P_{i1}^{*}(s_i) - E(c_i|s_i)\right), \tag{43}$$

where $P_{i1}^*(s_i)$ and $P_{j1}^*(s_j)$ are determined by (18) and (19). Take the derivative:

$$\frac{d\Pi_{i1}^*(E(c_i|s_i))}{dE(c_i|s_i)} = \frac{\partial\Pi_{i1}^*}{\partial E(c_i|s_i)} + \frac{\partial\Pi_{i1}^*}{\partial P_{i1}^*}\frac{\partial P_{i1}^*}{\partial E(c_i|s_i)}$$
$$= -\underbrace{\left(a - bP_{i1}^*(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^*(s_j)\right]\right)}_{q_{i1}^*} + \underbrace{\frac{\partial\Pi_{i1}^*}{\partial P_{i1}^*}}_{<0}\alpha_1^*, \tag{44}$$

where the second item in (44) follows from (18) with α_1^* defined in (20). In contrast to the non-signaling benchmark, now a change in $E(c_i|s_i)$ has a negative indirect impact on $\Pi_{i1}^*(E(c_i|s_i))$ through the pricing function P_{i1}^*. In other words, the Envelope Theorem does not hold at P_{i1}^*. This is because P_{i1}^* is distorted above the optimal non-signaling price which maximizes Firm i's static profit of the first period. When $E(c_i|s_i)$ increases, Firm i will raise P_{i1}^* by α_1^*, which will further increase the price distortion and reduce Firm i's first-period profit. So, the second item in (44) is negative.

Similar to the non-signaling benchmark, Firm i has a direct gain when $E(c_i|s_i)$ varies because it can adjust quantity optimally. Nevertheless, its direct gain is dampened by the indirect loss through P_{i1}^*. To see this, note that Firm i's profit is concave in its own price and P_{i1}^* is greater than the optimal non-signaling price. Thus, when P_{i1}^* varies, Firm i's loss from an increase in P_{i1}^* (when expected costs are high) outweighs its gain from a decrease in P_{i1}^* (when costs are low), which results in a loss in profit in expectation. Hence, the distortion in firms' first-period prices due to signaling incentives reduces the value of information.

In sum, both signaling and non-signaling firms benefit from information acquisition. However, signaling firms acquire less information than non-signaling firms, because the increased variation in prices caused by more accurate cost estimates harms the firm whose price is distorted above the static optimal prices due to signaling.

5.2 Welfare Implication of Signaling: Comparison to Non-Signaling Firms

Given the discussion on the social planer in Proposition 4 where it is shown that conditional on the firms' pricing strategies, firms choose inefficiently low levels of precision in the information acquisition decisions, Proposition 5 suggests that signaling leads to too little information acquisition from a social welfare perspective. However this supposition does not directly follow, because

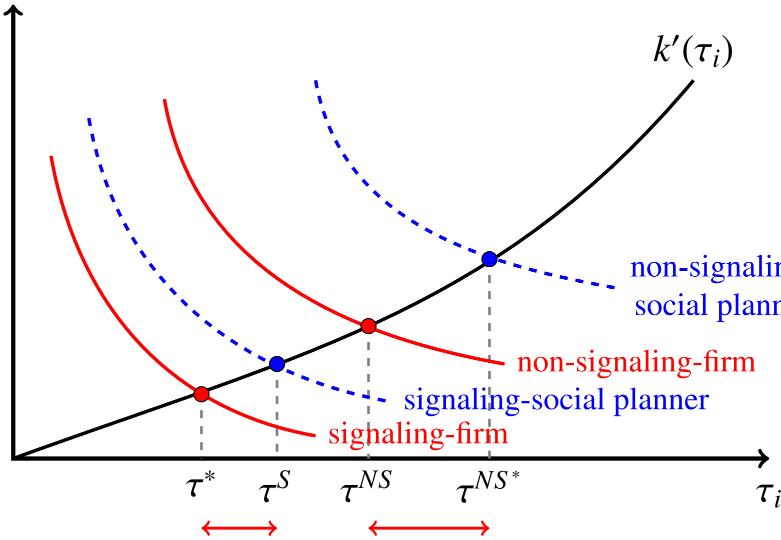

Figure 4: Efficiency of individual firms' investments

the welfare results from Section 4 are *conditioned on signaling strategies* being employed. When firms charge optimal non-signaling prices, the benchmark we use to evaluate the efficiency of non-signaling firms' information acquisition decisions should be adjusted according to the non-signaling prices.

To better understand the impact of signaling on the relative efficiency of information acquisition strategies, we wish to ascertain whether the discrepancy between individual firm's marginal benefit and the social marginal benefit from information acquisition is greater due to signaling when compared to the non-signaling benchmark. To do so, we calculate the discrepancy between the social benefit and individual firms' benefit from firms' investment in information acquisition in the case of non-signaling benchmarks and compare this to the case of signaling. The comparison is illustrated in Figure 4. The two downward sloping dashed lines are the social benefit from a firm's investment in information acquisition in both regimes, whereas the two downward sloping solid lines indicate the individual firm's benefit from its investment. Although signaling firms invest less in information acquisition than non-signaling firms, the gap between the social benefit and the individual firm's benefit is smaller for signaling firms than for non-signaling firms. In sum:

Proposition 6 *Signaling firms' investments in information acquisition is relatively more efficient than those of non-signaling firms.*

The discrepancy between the social benefit and an individual firm's benefit from information acquisition is larger for non-signaling firms than for signaling firms because non-signaling firms' information exerts a larger positive externality on consumer welfare. As is discussed in section 4.2, consumers benefit from variation in firms' quantities. Recall from Figure 2 that non-signaling firms' first period pricing strategy is steeper than that of signaling firms. So, compared with signaling firms, non-signaling firms' first-period prices experience a larger variation when they improve the precision of their signals, and this results in a larger variation in quantities.

6 Robustness

We briefly consider which findings are invariant to specific modeling assumptions. In particular, we first consider unobservable levels of investment in information acquisition rather than publicly observable investments. Secondly, we discuss quantity competition, rather than price competition. Lastly, we discuss demand uncertainty instead of cost uncertainty.

6.1 Non-observable investment

For expositional ease of the main analysis, we assumed that firms' investment decisions are publicly observable. Here we show that the equilibrium strategies presented in Propositions 1 and 2 in Section 3 are mutual best-replies also with non-observable investments, and therefore the strategies and beliefs (now including beliefs about the rival's unobservable investment strategy) are an equilibrium of the game with unobservable investment choices.

As the information acquisition decisions are no longer observable, let $\hat{\tau}_i^*$ denote Firm j's beliefs about the choice of precision of Firm i's signal τ_i^*. Taking as given Firm j's strategy and beliefs as derived in the equilibrium, with $\hat{\tau}_i^* \equiv \tau_i^*$ from Proposition 2, consider Firm i's best response assuming—in departure from the base model—that Firm i's investment decision is indeed not observed.

We first show that Firm i does not have a profitable deviation in its second-period price. Recall Firm j's equilibrium second-period price in (9), where the last two terms are Firm j's belief about Firm i's expected cost and Firm i's belief about Firm j's expected cost, respectively. Having fixed Firm j's strategies and beliefs, Firm j infers that Firm i's expected cost is $\frac{p_{i1}-\beta_1^*}{\alpha_1^*}$, and Firm i infers from j's equilibrium pricing strategy that Firm j's expected cost is $\frac{p_{j1}-\beta_1^*}{\alpha_1^*}$. Firm j's second-period price can be expressed as follows:

$$P_{j2}(c_j; p_{i1}, p_{j1}|\alpha_1^*, \beta_1^*) = \frac{a(2b+e)}{4b^2-e^2} + \frac{c_j}{2} + \frac{be}{4b^2-e^2}\frac{p_{i1}-\beta_1^*}{\alpha_1^*} + \frac{e^2}{2(4b^2-e^2)}\frac{p_{j1}-\beta_1^*}{\alpha_1^*}. \tag{45}$$

Using Firm i's first order condition (6), it can be verified that Firm i's best response is

$$P_{i2}(c_i; p_{i1}, p_{j1}|\alpha_1^*, \beta_1^*) = \frac{a(2b+e)}{4b^2-e^2} + \frac{c_i}{2} + \frac{be}{4b^2-e^2}\frac{p_{j1}-\beta_1^*}{\alpha_1^*} + \frac{e^2}{2(4b^2-e^2)}\frac{p_{i1}-\beta_1^*}{\alpha_1^*}. \tag{46}$$

Note that (45) is equivalent to (9) and (46) is equivalent to (8), after we use firms' equilibrium first-period pricing rules to substitute $\frac{p_{i1}-\beta_1^*}{\alpha_1^*}$ and $\frac{p_{j1}-\beta_1^*}{\alpha_1^*}$ for firms' beliefs about the rivals' expected cost $E(c_i|\hat{s}_i)$ and $E(c_j|\hat{s}_j)$ in (9) and (8).

The reason that Firm i does not deviate from its equilibrium second-period price when its investment is non-observable is because its actual investment does not affect the second-period competition. The only thing that matters in determining Firm i's second-period price is its rival's belief about i's investment, which does not change because the rival does not observe Firm i's deviation in investment, if there is any.

Moving to the first period, take as given τ_i and s_i. Firm i chooses p_{i1} to maximize the sum of profits from the two periods. The first order condition is

$$a + bE(c_i|s_i, \tau_i) - 2bp_{i1} + e\mathbb{E}_{s_j}\left[P_{j1}^*(s_j)|\tau_j^*\right] + \int_{s_j}\int_{c_i}\frac{\pi_{i2}^*(c_i, P_{i2}^*, P_{j2}^*)}{\partial p_{i1}}dF(c_i|s_i, \tau_i)dG_j(s_j|\tau_j^*) = 0. \tag{47}$$

Applying the Envelope Theorem and using Firm j's equilibrium second-period pricing function, the impact of p_{i1} on Firm i's second-period profit can be expressed as:

$$\begin{aligned}
&\int_{s_j}\int_{c_i}\frac{\pi_{i2}^*(c_i, P_{i2}^*, P_{j2}^*)}{\partial p_{i1}}dF(c_i|s_i, \tau_i)dG_j(s_j|\tau_j^*)\\
&\quad = \int_{s_j}\int_{c_i}\frac{\partial\pi_{i2}^*(c_i, P_{i2}^*, P_{j2}^*)}{\partial P_{j2}^*}\frac{\partial P_{j2}^*}{\partial p_{i1}}dF(c_i|s_i, \tau_i)dG_j(s_j|\tau_j^*)\\
&\quad = \frac{be^2}{(4b^2-e^2)\alpha_1^*}\left(\frac{a(2b+e)+be\mu_c}{4b^2-e^2} - \frac{E(c_i|s_i,\tau_i)}{2} + \frac{e^2}{2(4b^2-e^2)}\frac{p_{i1}-\beta_1^*}{\alpha_1^*}\right).
\end{aligned} \tag{48}$$

Substituting (48) into the first order condition (47), Firm i's optimal first-period price p_{i1} must satisfy

$$a + bE(c_i|s_i,\tau_i) - 2bp_{i1} + e\mathbb{E}_{s_j}\left[P^*_{j1}(s_j)|\tau^*_j\right] \\ + \frac{be^2}{(4b^2-e^2)\alpha^*_1}\left(\frac{a(2b+e)+be\mu_c}{4b^2-e^2} - \frac{E(c_i|s_i,\tau_i)}{2} + \frac{e^2}{2(4b^2-e^2)}\frac{p_{i1}-\beta^*_1}{\alpha^*_1}\right) = 0. \tag{49}$$

From (49) we can solve p_{i1} as an affine function of $E(c_i|s_i,\tau_i)$; and it can be verified that the slope of Firm i's pricing function is α^*_1 and the intercept is β^*_1.

Since Firm i adopts the same pricing strategies when its investment is observable and when it is non-observable to the rival, its marginal gain from investment remains the same as in (30). Hence, Firm i's optimal investment is, again, τ^* as defined in Proposition 2.

6.2 Demand uncertainty

The main findings are also robust to the source of uncertainty. Specifically, when firms face demand instead of cost uncertainty, (a) firms also distort prices above the optimal static level, (b) firms' expected total profits increase in the precisions of their signals, and (c) signaling reduces firms' value of information.

Under demand uncertainty, a firm has an incentive to distort its first-period price above the optimal static level in order to convince the rival that it faces a strong demand that will result in the firm continuing to charge a high price in the second period. In response, the rival charges a high second-period price in the case of substitutes and a low second-period price in the case of complements. In both cases, the rival increases the firm's residual demand and its second-period profit.

Similar to the case of cost uncertainty, more accurate demand forecasts help firms better attune their prices to demand and thus increase their expected first-period profits. In addition, more accurate demand forecasts also increase the correlation between the second-period prices (positive correlation for substitutes and negative correlation for complements). Take substitutes as an example. When a firm's demand forecast is more accurate, its prices are more positively correlated across the two periods. Thus, when the firm charges a high first-period price, its rival is more confident that the firm will charge a high price again in the second period and therefore will also price high in the second period. Different from the case of cost uncertainty, this positive price correlation increases the firm's second-period profit under demand uncertainty. To see why, note that the pass-through of demand to price is imperfect. So, when demand is high, a firm charges a high price and has a high profit margin, whereas when demand is low, the firm charges a low price and has a low profit margin. Positive price correlation implies that the rival firm adjusts its price in a way that increases the firm's residual demand at high profit margins and reduces residual demand at low profit margins, which results in a gain in the firm's expected second-period profit. The case of complementary goods follows analogously and firms' second-period profits increase in the degree of the negative correlation between their second-period prices. In summary, under demand uncertainty, firms benefit from more accurate forecasts of their market demands.

Lastly, firms' value of information is reduced by their signaling incentives. Under demand uncertainty, firms' first-period prices are linear in their expected underlying demand. More accurate demand forecasts increase the variance of firms' expected underlying demand conditional on the signals, which results in a larger variance in firms' first-period prices. While both signaling and non-signaling firms benefit from more accurate demand forecasts, signaling firms derive a smaller gain than non-signaling firms. This is because non-signaling firms price at the static optimal level and do not bear a cost from the price variation due to the Envelope Theorem. By contrast, because the profit function is concave in price and signaling firms price above the optimal static level, signaling firms bear a loss when adjusting prices in response to demand conditions.

6.3 Quantity competition

It is well-known that when switching from Bertrand (i.e., price-) competition to Cournot (i.e., quantity-) competition, many qualitative results are reversed. A case in point is how signaling with private information on costs plays out between the two modes of competition. Thus, in our setting firms distort prices above their non-signaling levels to suggest higher costs. The incentive to signal high costs is tied to prices being strategic complements: information on higher costs of one firm induce higher prices from the rival in the future, which softens competition and raises profits in the second period. In contrast, in Cournot competition with private cost information, signaling leads to firms increasing their output above non-signaling levels in order to suggest lower costs. Due to outputs being strategic substitutes a firm's low costs and high output serve the (futile) purpose to have the rival reduce output, thereby softening competition and increasing profits. An implication of the marked difference in the method of signaling across the two modes (feigning high costs in Bertrand competition v. feigning low costs in Cournot competition) is that whereas in our framework signaling is anticompetitive, quantity signaling is actually welfare enhancing, because it lowers prices, bringing them closer to the perfect competitive level.

Nevertheless, our main result that signaling reduces the value of information holds across both modes of competition. The logic for the case of quantity competition is analogous to that in the price signaling model. When firms invest in information acquisition, their signals about costs become more precise. This is to their benefit in terms of first period profit as it brings their choice variable (price or output) more in line with the true underlying state of the world. However, because in both models of competition signaling incentives result in an upward distortion in firms' first-period strategic variables above the non-signaling choices, over-shooting is more costly than under-shooting, and firms refrain obtaining as much information as they otherwise would in order to reduce the variance of their choices. Thus, signaling incentives—wether in price or quantity competition—reduce the value of information and lead to firms acquiring less precise signals about their costs when compared to non-signaling firms.

7 Conclusion

We study firms' incentives to acquire private information when they anticipate that they signal private information to their rivals through their prices. Overall, firms benefit when the qualities of their private information are improved. However, "signaling" reduces firms' incentives to acquire more accurate information. Compared to non-signaling firms, signaling firms will thus acquire less precise signals.

From the perspective of the industry, the qualities of firms' information are inefficiently low, which is driven by a positive externality of firms' improved information on their rivals' second-period profits. When firms acquire more accurate information, consumers benefit in the first period, but suffer a loss in the second period. Overall, firms' more accurate information has a positive effect on consumers when the degree of substitution or complementarity between the goods is not too high.

From the social planner's perspective, the qualities of firms' private information are also inefficiently low. As a result, it may be preferable from a social planer's perspective to allow a trade association to acquire information on the firms' behalf, even when the trade association acquires too much information from a total welfare perspective.

A Appendix

Proof of Lemma 3.1 The first order conditions, (6) and (7), imply that each firm's second-period price function is affine in its own cost. Using

$$P_{i2}(c_i; \hat{s}_i, \hat{s}_j) = \alpha_{i2} c_i + \beta_{i2} \tag{50}$$

$$P_{j2}(c_j; \hat{s}_j, \hat{s}_i) = \alpha_{j2} c_j + \beta_{j2}, \tag{51}$$

together with (6) and (7), we solve for the unique set of parameters

$$\begin{aligned} \alpha_{i2} = \alpha_{j2} &= \frac{1}{2}, \\ \beta_{i2} &= \frac{a(2b+e)}{4b^2 - e^2} + \frac{beE(c_j|\hat{s}_j)}{4b^2 - e^2} + \frac{e^2 E(c_i|\hat{s}_i)}{2(4b^2 - e^2)}, \\ \beta_{j2} &= \frac{a(2b+e)}{4b^2 - e^2} + \frac{beE(c_i|\hat{s}_i)}{4b^2 - e^2} + \frac{e^2 E(c_j|\hat{s}_j)}{2(4b^2 - e^2)}. \end{aligned}$$

□

Proof of Proposition 1 In proving the proposition, we first prove a lemma about a necessary property of any revealing (i.e., separating) equilibrium in our setting concerning the generic form of the differential equation that the first period pricing rule must satisfy. Thereafter we prove that any linear pricing rule is unique; and finally we show that there does not exist a non-linear pricing rule satisfying the necessary condition on the differential equation.

To lay the groundwork, define $x_i := E(c_i|s_i)$. Given (4), there is a one-to-one mapping between s_i and $E(c_i|s_i)$. It is without of loss of generality to regard Firm i's first-period pricing rule as a function of x_i. Assume that firms' first period pricing functions are differentiable. It follows that,

$$P'_{i1}(s_i) = P'_{i1}(x_i(s_i)) x'_i(s_i) = P'_{i1}(x_i) \bar{\tau}_i. \tag{52}$$

Lemma A.1 *In any fully revealing equilibrium, Firm i's first-period price rule $P_{i1}(x_i)$ must satisfy the following differential equation:*

$$P'_{i1}(x_i)\,(M_3 + x_i - 2P_{i1}(x_i)) = M_2 x_i - M_1, \tag{53}$$

where M_1, M_2, M_3 are constants; and

$$P_{i1}(x_{i0}) = \frac{M_3}{2} + \frac{M_1}{2M_2}, \ \textit{for} \ \ x_{i0} = \frac{M_1}{M_2}, \ \textit{and} \tag{54}$$

$$\frac{e^4}{4(4b^2 - e^2)^2} < [P'_{i1}(x_i)]^2. \tag{55}$$

Proof of Lemma A.1. We first show that any fully revealing equilibrium must satisfy the differential equation (53). Substitute P_{i2} defined in (8) into (16), Firm i's gain from price distortion in the first period is

$$\begin{aligned}
\frac{\partial \Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}} &= \frac{be^2}{(4b^2 - e^2)} \frac{\bar{\tau}_i}{P'_{i1}(s_i)} \int_{c_i} \int_{\hat{s}_j} \left(\frac{a(2b + e)}{4b^2 - e^2} - \frac{c_i}{2} + \frac{beE(c_j|\hat{s}_j)}{4b^2 - e^2} + \frac{e^2 E(c_i|\hat{s}_i)}{2(4b^2 - e^2)} \right) dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \frac{be^2}{(4b^2 - e^2)} \frac{\bar{\tau}_i}{P'_{i1}(s_i)} \left(\frac{a(2b + e) + be\mu_c}{4b^2 - e^2} + \frac{e^2 - 2b^2}{4b^2 - e^2} E(c_i|s_i) \right),
\end{aligned} \tag{56}$$

where the second equality is obtained after imposing consistent beliefs $s_i = \hat{s}_i$ and $s_j = \hat{s}_j$. Using $P'_{i1}(s_i) = P'_{i1}(x_i)\bar{\tau}_i$ and substituting (56) and $E(c_i|s_i) = x_i$ into (14), Firm i's first order condition can be written as follows:

$$a + bx_i - 2bP_{i1}(x_i) + e\mathbb{E}_{x_j}(P_{j1}(x_j)) + \frac{be^2}{4b^2 - e^2} \frac{1}{P'_{i1}(x_i)} \left(\frac{a(2b + e) + be\mu_c}{4b^2 - e^2} + \frac{e^2 - 2b^2}{4b^2 - e^2} x_i \right) = 0$$

which can be written as

$$P'_{i1}(x_i)\,[M_3 + x_i - 2P_{i1}(x_i)] = M_2 x_i - M_1.$$

where[11]

$$M_3 = \frac{a + e\mathbb{E}_{x_j}(P_{j1}(x_j))}{b}, \qquad M_2 = \frac{e^2(2b^2 - e^2)}{(4b^2 - e^2)^2}, \qquad M_1 = \frac{e^2(a(2b + e) + be\mu_c)}{(4b^2 - e^2)^2}. \tag{57}$$

Next, we show that any fully revealing equilibrium must satisfy the initial condition (54). By strict monotonicity, $P'_{i1}(x_i) \neq 0, \forall x_i$. Hence, for the differential equation to hold at $x_i = x_{i0}$, it is necessary that the expression in the bracket on the left hand side of (53) is zero, which implies $P_{i1}(x_{i0}) = \frac{M_3}{2} + \frac{M_1}{2M_2}$.

Condition (55) ensures that the first order condition is sufficient. To see this, following (14), Firm i's second order condition is

$$-2b + \frac{\partial^2 \Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}^2} < 0. \tag{58}$$

Note further that

$$\begin{aligned}
\frac{\partial^2 \Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}^2} &= \frac{be^2}{4b^2 - e^2} \frac{\bar{\tau}_i}{P'_{i1}(s_i)} \int_{c_i} \int_{\hat{s}_j} \frac{\partial P_{i2}}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial p_{i1}} dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \frac{be^4 \bar{\tau}_i}{2(4b^2 - e^2)^2} \frac{1}{P'_{i1}(s_i)} \int_{c_i} \int_{\hat{s}_j} \left(\frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial p_{i1}} \right) dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \frac{be^4}{2(4b^2 - e^2)^2} \left(\frac{\bar{\tau}_i}{P'_{i1}(s_i)} \right)^2 \\
&= \frac{be^4}{2(4b^2 - e^2)^2} \left(\frac{1}{P'_{i1}(x_i)} \right)^2.
\end{aligned} \tag{59}$$

The first equality follows from the first line in (56), the second equality follows from (8), and the third equality follows from (4) as well as the inverse function theorem.

Substitute (59) into (58) to obtain

$$\frac{e^4}{4(4b^2 - e^2)^2} < \left[P'_{i1}(x_i)\right]^2. \tag{60}$$

□

The differential equation (53) follows from Firm i's first order condition (14), and (55) ensures that

[11] Note that while M_3 is a function of $\mathbb{E}_{x_j}(P_{j1}(x_j))$, since $\mathbb{E}_{x_j}(P_{j1}(x_j))$ does not involve P_{i1} or x_i, it is a constant from Firm i's perspective.

the first order condition is necessary and sufficient. Any fulling revealing equilibrium must satisfy the initial condition in (54). To see this, strict monotonicity requires $P'_{i1}(x_i) \neq 0, \forall x_i$. At x_{i0}, the right hand side of the differential equation is zero which implies that the term in the bracket on the left hand side must be zero. Thus, it follows that $P_{i1}(x_{i0}) = \frac{M_3}{2} + \frac{M_1}{2M_2}$.

Given Lemma A.1, the proof of the proposition has two steps. Step 1 shows that there is a unique linear equilibrium. Step 2 shows that there does not exist a nonlinear fully revealing equilibrium.

Step 1. Consider arbitrary linear first period pricing strategies

$$P_{i1}(x_i) = \alpha_{i1} x_i + \beta_{i1} \tag{61}$$

$$P_{j1}(x_j) = \alpha_{j1} x_j + \beta_{j1}. \tag{62}$$

In a linear equilibrium, Firm i's price $P_{i1}(x_i)$ must satisfy (53):

$$\begin{aligned} \alpha_{i1}\left[M_3 - 2(\alpha_{i1} x_i + \beta_{i1}) + x_i\right] &= M_2 x_i - M_1 \\ \alpha_{i1}(1-2\alpha_{i1})x_i + \alpha_{i1}(M_3 - 2\beta_{i1}) &= M_2 x_i - M_1 \\ \alpha_{i1}(1-2\alpha_{i1})x_i + \alpha_{i1}\left(\frac{a + e(\alpha_{j1}\mu_c + \beta_{j1})}{b} - 2\beta_{i1}\right) &= M_2 x_i - M_1. \end{aligned} \tag{63}$$

We obtain the last equality by substituting (62) into M_3 defined in (57). By symmetry, Firm j's pricing rule $P_{j1}(x_j)$ satisfies

$$\alpha_{j1}(1-2\alpha_{j1})x_j + \alpha_{j1}\left(\frac{a + e(\alpha_{i1}\mu_c + \beta_{i1})}{b} - 2\beta_{j1}\right) = M_2 x_j - M_1. \tag{64}$$

The parameters α_{i1}, β_{i1}, α_{j1} and β_{j1} must solve the following system of equations:

$$\alpha_{i1}(1-2\alpha_{i1}) = M_2 \tag{65}$$

$$\alpha_{i1}\left(\frac{a + e(\alpha_{j1}\mu_c + \beta_{j1})}{b} - 2\beta_{i1}\right) = -M_1 \tag{66}$$

$$\alpha_{j1}(1-2\alpha_{j1}) = M_2 \tag{67}$$

$$\alpha_{j1}\left(\frac{a + e(\alpha_{i1}\mu_c + \beta_{i1})}{b} - 2\beta_{j1}\right) = -M_1, \tag{68}$$

where M_1 and M_2 are defined in (57). There are two sets of solutions $\{\alpha_{i1}, \beta_{i1}, \alpha_{j1}, \beta_{j1}\}$ for the above system of equations. However, the initial condition is satisfied and the first order condition is sufficient only at the solution specified in Proposition 1. To see this, given $0 < |e| < b$, the second order condition (55) is satisfied at $\alpha_{i1} = \alpha_{j1} = \frac{2b^2-e^2}{4b^2-e^2}$, but is violated at the other root $\alpha_{i1} = \alpha_{j1} = \frac{e^2}{2(4b^2-e^2)}$.

Step 2. Totally differentiating the differential equation (53) gives,

$$P''_{i1}(x_i)\,[M_3 + x_i - 2P_{i1}(x_i)] + P'_{i1}(x_i)\,[1 - 2P'_{i1}(x_i)] = M_2, \tag{69}$$

Evaluate (69) at the initial value $x_i = x_{i0}$, which implies $M_3 + x_{i0} - 2P_{i1}(x_{i0}) = 0$ and yields

$$2[P'_{i1}(x_{i0})]^2 - P'_{i1}(x_{i0}) + M_2 = 0$$
$$P'_{i1}(x_{i0}) = \frac{1 \pm \sqrt{1 - 8M_2}}{4}. \tag{70}$$

Totally differentiating (69), yields

$$P'''_{i1}(x_i)\,[M_3 + x_i - 2P_{i1}(x_i)] + 2P''_{i1}(x_i)\,[1 - 3P'_{i1}(x_i)] = 0 \tag{71}$$

Again, evaluate (71) at x_{i0}, it follows that

$$P''_{i1}(x_{i0})\,[1 - 3P'_{i1}(x_{i0})] = 0. \tag{72}$$

Given $0 < |e| < b$, $P'_{i1}(x_{i0})$ in (70) is strictly less than $1/3$, and hence (71) implies $P''_{i1}(x_{i0}) = 0$.

Repeat the same process, use the initial condition and $P_{i1}^{(n-1)}(x_{i0}) = 0, \forall n = 3, 4, 5...$, we obtain

$$P_{i1}^{(n)}(x_{i0})\,[n - 2(1+n)P'_{i1}(x_{i0})] = 0. \tag{73}$$

Because $P'_{i1}(x_{i0}) < \frac{n}{2(1+n)}$, $P_{i1}^{(n)}(x_{i0}) = 0$. Using a Taylor expansion, Firm i's first-period price function can be written as

$$P_{i1}(x_i) = \sum_{n=0}^{\infty} \frac{P_{i1}^{(n)}(x_{i0})}{n!}(x_i - x_{i0})^n. \tag{74}$$

Given that $P_{i1}^{(n)}(x_{i0}) = 0, \forall n = 2, 3, 4..., P_{i1}(x_i)$ is linear in x_i. $\square$

Proof of Lemma 3.2 Consider a general linear pricing rule $P_{i1}(s_i) = \alpha_1 E(c_i|s_i) + \beta_1$. Recalling (11), conditional on signal s_i, Firm i's expected first-period profit is

$$\begin{aligned}\Pi_{i1}(s_i) &= \left(a - bP_{i1}(s_i) + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\right)(P_{i1}(s_i) - E(c_i|s_i)) \\ &= -b(P_{i1}(s_i))^2 + bP_{i1}(s_i)E(c_i|s_i) + \left(a + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\right)(P_{i1}(s_i) - E(c_i|s_i))\end{aligned} \tag{75}$$

Taking expectations over s_i, Firm i's *ex ante* expected first-period profit is

$$\mathbb{E}_{s_i}\Pi_{i1}(s_i) = -b\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] + b\mathbb{E}_{s_i}\left[P_{i1}(s_i)E(c_i|s_i)\right] + \left(a + e\mathbb{E}_{s_j}\left[P_{j1}(s_j)\right]\right)\left(\mathbb{E}_{s_i}\left[P_{i1}(s_i)\right] - \mu_c\right). \tag{76}$$

The last item in (76) does not involve τ_i or τ_j; and using the general linear pricing rule, the second item in (76) can be written as

$$\begin{aligned}b\mathbb{E}_{s_i}\left[P_{i1}(s_i)E(c_i|s_i)\right] &= b\mathbb{E}_{s_i}\left[P_{i1}(s_i)\left(\frac{P_{i1}(s_i)}{\alpha_1} - \frac{\beta_1}{\alpha_1}\right)\right] \\ &= \frac{b}{\alpha_1}\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] - \frac{b\beta_1}{\alpha_1}\mathbb{E}_{s_i}\left[P_{i1}(s_i)\right],\end{aligned} \tag{77}$$

where $\mathbb{E}_{s_i}[P_{i1}(s_i)] = \alpha_1\mu_c + \beta_1$ does not involve τ_i. Collecting all terms not involving τ_i or τ_j into Ψ we have

$$\begin{aligned}\mathbb{E}_{s_i}\Pi_{i1}(s_i) &= b\left(\frac{1}{\alpha_1} - 1\right)\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] + \Psi \\ &= b\left(\frac{1}{\alpha_1} - 1\right)\mathrm{Var}(P_{i1}(s_i)) + \Psi \\ &= b\alpha_1(1-\alpha_1)\mathrm{Var}(E(c_i|s_i)) + \Psi \\ &= b\alpha_1(1-\alpha_1)\frac{\tau_i}{(\tau_c + \tau_i)\tau_c} + \Psi,\end{aligned} \tag{78}$$

where the last equation is due to the following:

$$\mathrm{Var}(E(c_i|s_i)) = (\bar{\tau}_i)^2\mathrm{Var}(s_i) = (\bar{\tau}_i)^2\left(\mathbb{E}(\mathrm{Var}(s_i|c_i)) + \mathrm{Var}(\mathbb{E}(s_i|c_i))\right) = \frac{\tau_i}{\tau_c(\tau_i + \tau_c)}, \tag{79}$$

the first equality follows from (4), the second equality follows from the law of total variance, and the last equality is obtained after substituting $\bar{\tau}_i$ defined in (4).

Thus, Firm i's marginal gain from an improvement in the quality of its signal is

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}(s_i)}{\partial \tau_i} = b\alpha_1(1-\alpha_1)\frac{1}{(\tau_c+\tau_i)^2}. \tag{80}$$

In equilibrium, $\alpha_1 = \alpha_1^*$ and $\beta_1 = \beta_1^*$, which are defined by (20) and (21), respectively.

After substituting $\alpha_1 = \alpha_1^*$ into (80), Firm i's marginal gain in its expected equilibrium first-period profit is:

$$\frac{\partial \mathbb{E}_{s_i} \Pi^*_{i1}(s_i)}{\partial \tau_i} = \frac{2b^3(2b^2-e^2)}{(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2}. \tag{81}$$

By assumption $b > |e|$, so $\partial \mathbb{E}_{s_i} \Pi^*_{i1}(s_i)/\partial \tau_i > 0$. □

Proof of Lemma 3.3 For notational convenience, express firms' second-period equilibrium prices (8) and (9) as

$$P^*_{i2}(c_i, s_i, s_j) = z_0 + \frac{c_i}{2} + z_1 E(c_j|s_j) + z_2 E(c_i|s_i), \tag{82}$$

$$P^*_{j2}(c_j, s_j, s_i) = z_0 + \frac{c_j}{2} + z_1 E(c_i|s_i) + z_2 E(c_j|s_j), \tag{83}$$

where $z_0 = \frac{a(2b+e)}{4b^2-e^2}$, $z_1 = \frac{be}{4b^2-e^2}$, and $z_2 = \frac{e^2}{2(4b^2-e^2)}$.

Using the first order condition, Firm i's expected second period equilibrium profit is

$$\Pi^*_{i2}(s_i) = \mathbb{E}_{s_j}\mathbb{E}_{c_i} b[P^*_{i2}(c_i, s_i, s_j) - c_i]^2 \tag{84}$$

$$\mathbb{E}_{s_i}\Pi^*_{i2}(s_i) = \mathbb{E}_{s_i}\mathbb{E}_{s_j}\mathbb{E}_{c_i} b\left[z_0 + \frac{c_i}{2} + z_1 E(c_j|s_j) + z_2 E(c_i|s_i) - c_i\right]^2 \tag{85}$$

$$\begin{aligned} &= b\mathrm{Cov}\left(\frac{1}{2}c_i, z_2 E(c_i|s_i)\right) + b\mathrm{Var}\left(z_2 E(c_i|s_i)\right) + b\mathrm{Cov}\left(z_2 E(c_i|s_i), -c_i\right) \\ &\quad + b\mathrm{Var}\left(z_1 E(c_j|s_j)\right) + \Psi \end{aligned} \tag{86}$$

$$\begin{aligned} &= \frac{e^2 b}{4(4b^2-e^2)}\mathrm{Var}(E(c_i|s_i)) + \frac{e^4 b}{4(4b^2-e^2)^2}\mathrm{Var}(E(c_i|s_i)) \\ &\quad - \frac{e^2 b}{2(4b^2-e^2)}\mathrm{Var}(E(c_i|s_i)) + \frac{b^3 e^2}{(4b^2-e^2)^2}\mathrm{Var}(E(c_j|s_j)) + \Psi \end{aligned} \tag{87}$$

$$= -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau_i}{\tau_c(\tau_c+\tau_i)} + \frac{b^3e^2}{(4b^2-e^2)^2}\frac{\tau_j}{\tau_c(\tau_c+\tau_j)} + \Psi \tag{88}$$

In the derivation, we collect all terms that do not involve signal precision into Ψ. In obtaining (86), we've used the assumption that $c_i,\ c_j$ are independent. (87) follows from the fact $\mathbb{E}(c_i, E(c_i|s_i)) = \mathrm{Var}(E(c_i|s_i)) + \text{constant}$, where $\mathrm{Var}(E(c_2|c_1))$ given by (79).

It can be verified that

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^*(s_i)}{\partial \tau_i} = -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{1}{(\tau_i+\tau_c)^2} < 0 \text{ and}$$
$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^*(s_i)}{\partial \tau_j} = \frac{b^3e^2}{(4b^2-e^2)^2}\frac{1}{(\tau_j+\tau_c)^2} > 0.$$

□

Proof of Proposition 2 It can be verified that the left hand side of (31) is strictly decreasing and that $\lim_{\tau_i\to 0}\frac{b(4b^2-e^2)(4b^2-3e^2)}{4(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2} = +\infty$ whereas $\lim_{\tau_i\to +\infty}\frac{b(4b^2-e^2)(4b^2-3e^2)}{4(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2} = 0$. On the other hand, by assumption $k'(\cdot)$ is strictly increasing and continuous, $k'(0) = 0$ and $\lim_{\tau_i\to+\infty} k'(\tau_i) = +\infty$, hence there's a unique solution to (31). □

Proof of Proposition 3 The trade association's objective is to choose signal precisions to maximize the sum of the firms' non-cooperative market profits net of the information acquisition costs; i.e., $\left(\tau_i^{TA}, \tau_j^{TA}\right) = \mathrm{argmax}_{\tau_i,\tau_j\geq 0}\sum_{t=1}^2 TP_t^* - k(\tau_i) - k(\tau_j)$, with $TP_t^* = \mathbb{E}_{s_i}\Pi_{it}^*(s_i) + \mathbb{E}_{s_j}\Pi_{jt}^*(s_j)$.

To show that the trade association increases the information acquisition, it suffices to show that

$$\begin{aligned}\frac{\partial \sum_{t=1}^2 TP_t^*}{\partial \tau_i} &= \frac{\partial TP_1^*}{\partial \tau_i} + \frac{\partial TP_2^*}{\partial \tau_i} = \frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^*(s_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^*(s_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_j}\Pi_{j2}^*(s_j)}{\partial \tau_i}\\ &> \frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^*(s_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^*(s_i)}{\partial \tau_i},\end{aligned} \tag{89}$$

where the inequality follows from $\partial\mathbb{E}_{s_j}\Pi_{j2}^*(s_j)/\partial\tau_i > 0$ (see Lemma 3.3). Note that the right hand side of the inequality is Firm i's marginal gain from improved signal precision. So $\tau_i^{TA} > \tau_i^*$, and symmetrically for Firm j.

Using (32) and (33) and applying (25) and (28) from Lemmata 3.2 and 3.3 yields,

$$\begin{aligned}\frac{\partial \sum_{t=1}^2 TP_t^*}{\partial \tau_i} &= \frac{\partial TP_1^*}{\partial \tau_i} + \frac{\partial TP_2^*}{\partial \tau_i}\\ &= \frac{b\left[\left(4b^2-e^2\right)\left(4b^2-3e^2\right)+4b^2e^2\right]}{4(4b^2-e^2)^2}\frac{1}{(\tau_i+\tau_c)^2} > 0,\end{aligned} \tag{90}$$

and so τ_i^{TA} is uniquely determined by

$$\frac{b\left[\left(4b^2-e^2\right)\left(4b^2-3e^2\right)+4b^2e^2\right]}{4(4b^2-e^2)^2}\frac{1}{(\tau_i+\tau_c)^2}=k'(\tau_i). \tag{91}$$

With Firm j's acquisition level being determined analogously. □

Proof of Proposition 4 The proof has two steps. Step 1 gives the social planner's optimal choice of investment in information acquisition. Step 2 compares the social planner's investment with the individual firms' independent investment decisions and the trade association's investment, respectively.

Step 1. Using demand functions (2) and (3), expected equilibrium social welfare in period t is

$$\begin{aligned}\mathbb{E}\left[SW_t^*\right]=&\left(-\frac{\eta_1}{2}(b^2+e^2)+\eta_2 be\right)\left(\mathrm{Var}(P_{it}^*)+\mathrm{Var}(P_{jt}^*)\right)+\left(2\eta_1 be-\eta_2(b^2+e^2)\right)\mathrm{Cov}(P_{it}^*,P_{jt}^*)\\&+b\left(\mathrm{Cov}(c_i,P_{it}^*)+\mathrm{Cov}(c_j,P_{jt}^*)\right)-e\left(\mathrm{Cov}(c_i,P_{jt}^*)+\mathrm{Cov}(c_j,P_{it}^*)\right)+\Psi.\end{aligned} \tag{92}$$

First consider $\mathbb{E}\left[SW_1^*\right]$. Because c_i and c_j are independent, first-period prices are independent; and each firm's cost is also independent of its rival's first-period price. Hence, $\mathrm{Cov}(P_{i1}^*,P_{j1}^*)=\mathrm{Cov}(c_i,P_{j1}^*)=\mathrm{Cov}(c_j,P_{i1}^*)=0$.

Following from (18) and (19), we have $\mathrm{Var}(P_{i1}^*)=(\alpha_1^*)^2\mathrm{Var}(E(c_i|s_i))$ and $\mathrm{Cov}(c_i,P_{i1}^*)=\alpha_1^*\mathrm{Var}(E(c_i|s_i)$; and the terms $\mathrm{Var}(P_{j1}^*)$ and $\mathrm{Cov}(c_j,P_{j1}^*)$ are obtained by symmetry. Substituting the variance in prices and the covariance between costs and prices, we obtain

$$\mathbb{E}\left[SW_1^*\right]=\left(\left(-\frac{\eta_1}{2}(b^2+e^2)+\eta_2 be\right)(\alpha_1^*)^2+b\alpha_1^*\right)\left(\mathrm{Var}(E(c_i|s_i))+\mathrm{Var}(E(c_j|s_j))\right)+\Psi \tag{93}$$

$$=\frac{(6b^2-e^2)(2b^2-e^2)b}{2(4b^2-e^2)^2}\left(\mathrm{Var}(E(c_i|s_i))+\mathrm{Var}(E(c_j|s_j))\right)+\Psi, \tag{94}$$

where (94) is obtained by substituting $\alpha_1^*=\frac{2b^-e^2}{4b^2-e^2}$, $\eta_1=\frac{b}{b^2-e^2}$, $\eta_2=\frac{e}{b^2-e^2}$; where η_1 and η_2 are derived from $a=\frac{\eta_0}{\eta_1+\eta_2}, b=\frac{\eta_1}{\eta_1^2-\eta_2^2}, e=\frac{\eta_2}{\eta_1^2-\eta_2^2}$.

Next, we derive $\mathbb{E}\left[SW_2^*\right]$. Recall that firms' second-period equilibrium prices can be expressed as (82) and (83) and so using

$$\text{Var}(P_{i2}^*) = z_1^2 \text{Var}(E(c_j|s_j)) + \left(z_2^2 + z_2\right) \text{Var}(E(c_i|s_i)) + \Psi \tag{95}$$

$$\text{Var}(P_{j2}^*) = z_1^2 \text{Var}(E(c_i|s_i)) + \left(z_2^2 + z_2\right) \text{Var}(E(c_j|s_j)) + \Psi \tag{96}$$

$$\text{Cov}(P_{i2}^*, P_{j2}^*) = \left(\frac{z_1}{2} + z_1 z_2\right)\left(\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))\right) + \Psi \tag{97}$$

$$\text{Cov}(c_i, P_{i2}^*) = z_2 \text{Var}(E(c_i|s_i)) \quad \text{and} \quad \text{Cov}(c_j, P_{j2}^*) = z_2 \text{Var}(E(c_j|s_j) \tag{98}$$

$$\text{Cov}(c_i, P_{j2}^*) = z_1 \text{Var}(E(c_i|s_i)) \quad \text{and} \quad \text{Cov}(c_j, P_{i2}^*) = z_1 \text{Var}(E(c_j|s_j); \tag{99}$$

to substitute into (92), we obtain

$$\begin{aligned}
\mathbb{E}\left[SW_2^*\right] = &\left\{\left[-\frac{\eta_1}{2}\left(b^2 + e^2\right) + \eta_2 be\right]\left(z_2^2 + z_1^2 + z_2\right) + 2\eta_1 be - \eta_2\left(b^2 + e^2\right)\left(\frac{z_1}{2} + z_1 z_2\right) + bz_2 - ez_1\right\} \times \\
&\times \left(\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))\right) + \Psi \qquad (100) \\
= &-\frac{\left(12b^2 - 5e^2\right) be^2}{8\left(4b^2 - e^2\right)^2}\left(\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))\right) + \Psi, \qquad (101)
\end{aligned}$$

where (101) is obtained by substituting the expressions for η_1, η_2 as well as z_1, z_2 as defined following (82) and (83).

Using (94) and (101), the total social welfare from the two periods is

$$\begin{aligned}
\sum_{t=1}^{2} \mathbb{E}\left[SW_t^*\right] &= \frac{b\left(48b^4 - 44b^2e^2 + 9e^4\right)}{8\left(4b^2 - e^2\right)^2}\left(\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))\right) + \Psi \qquad (102) \\
&= \frac{b\left(48b^4 - 44b^2e^2 + 9e^4\right)}{8\left(4b^2 - e^2\right)^2}\left(\frac{\tau_i}{\tau_c(\tau_i + \tau_c)} + \frac{\tau_j}{\tau_c(\tau_j + \tau_c)}\right) + \Psi, \qquad (103)
\end{aligned}$$

where the second equality follows from (79).

The social planner's optimal choices of investment in information acquisition is symmetric and is uniquely determined by

$$\frac{b\left(48b^4 - 44b^2e^2 + 9e^4\right)}{8\left(4b^2 - e^2\right)^2} \frac{1}{(\tau_i + \tau_c)^2} = k'(\tau_i).$$

Step 2. We first compare the social planner's investment choice with an individual firm's investment choice. The difference between the social planner's marginal gain from an increase in τ_i and Firm i's marginal gain (refer to (31)) is

$$\begin{aligned}
\frac{\partial \sum_{t=1}^{2} \mathbb{E}\left[SW_t^*\right]}{\partial \tau_i} - \frac{\partial \sum_{t=1}^{2} \mathbb{E}_{s_i}\Pi_{it}^*(s_i)}{\partial \tau_i} &= \left(\frac{b\left(48b^4 - 44b^2e^2 + 9e^4\right)}{8\left(4b^2 - e^2\right)^2} - \frac{b\left(4b^2 - 3e^2\right)}{4(4b^2 - e^2)}\right)\frac{1}{(\tau_i + \tau_c)^2} \\
&= \frac{b\left(16b^4 - 12b^2e^2 + 3e^4\right)}{8\left(4b^2 - e^2\right)^2}\frac{1}{(\tau_i + \tau_c)^2} \qquad (104) \\
&> \frac{b\left(4b^4 + 3e^4\right)}{8\left(4b^2 - e^2\right)^2}\frac{1}{(\tau_i + \tau_c)^2} > 0. \qquad (105)
\end{aligned}$$

This implies that the social planner prefers more information acquisition than Firm i.

Next, compare the social planner's investment with the trade association. Using (34), we derive

$$\frac{\partial \sum_{t=1}^{2} \mathbb{E}\left[SW_t^*\right]}{\partial \tau_i} - \frac{\partial \sum_{t=1}^{2} TP_t^*}{\partial \tau_i} = \frac{b\left(16b^4 - 20b^2e^2 + 3e^4\right)}{8\left(4b^2 - e^2\right)^2}\frac{1}{(\tau_i + \tau_c)^2}, \qquad (106)$$

which is negative when $|e|$ is sufficiently close to b and positive, otherwise. So, the social planner desires less information than the trade association when the goods are closely related and more information when they are more independent. □

Proof of Proposition 5 We first prove that there is a unique solution for non-signaling firms' optimal information acquisition problem. Denote by τ^{NS} Firm i's signal precision in the non-signaling benchmark. Since non-signaling firms' first-period prices are affine in their conditional expectation of costs (refer to (22) and (23)), we can use the proof of Lemma 3.2 which shows that when Firm i's first-period price is affine in $E(c_i|s_i)$, the firm's marginal gain from an improvement in its private information is (80). Substitute $\alpha_1 = \frac{1}{2}$ into (80), the non-signaling firm's first-period gain from a marginal improvement in its private information is

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)}{\partial \tau_i} = \frac{b}{4}\frac{1}{(\tau_c + \tau_i)^2}. \qquad (107)$$

Because the expected equilibrium second-period profits are the same for signaling and non-signaling firms, it follows that

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^{NS}(s_i)}{\partial \tau_i} = \frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^{*}(s_i)}{\partial \tau_i},$$

which is given by (27). Thus, using (107) and (27)

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i}^{NS}(s_i)}{\partial \tau_i} = \frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^{NS}(s_i)}{\partial \tau_i} = \frac{b(4b^2-2e^2)^2}{4(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2} > 0. \tag{108}$$

Firm i's optimal signal precision is the solution to

$$k'(\tau_i) = \frac{\partial \mathbb{E}_{s_i}\Pi_{i}^{NS}(s_i)}{\partial \tau_i} = \frac{b(4b^2-2e^2)^2}{4(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2}. \tag{109}$$

Note that $\frac{\partial \mathbb{E}_{s_i}\Pi_{i}^{NS}(s_i)}{\partial \tau_i}$ is decreasing in τ_i, positive at $\tau_i = 0$ and approaching 0 as $\tau_i \to +\infty$. Given the assumptions $k''(\cdot) > 0$, $k'(0) = 0$ and $\lim_{\tau_i \to +\infty} k'(\tau_i) = \infty$, there exists a unique solution $\tau^{NS} \in (0, \infty)$ to Equation (109).

To compare information acquisition decisions of signaling and non-signaling firms, it suffices to compare the marginal gain in firms' first-period profits from an improvement in the quality of their private information:

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)}{\partial \tau_i} - \frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{*}(s_i)}{\partial \tau_i} = \frac{be^4}{4(4b^2-e^2)}\frac{1}{(\tau_i+\tau_c)^2} > 0, \tag{110}$$

which implies $\tau^* < \tau^{NS}$. Since (110) increases in $|e|$, the degree of substitution between the two goods, the difference in signal precisions $\tau^{NS} - \tau^*$ increases in $|e|$. □

Proof of Proposition 6 We begin by calculating the social planner's optimal level of investment when facing non-signaling firms. Similar to the case of signaling firms, the social planner's expected payoff in period t can be expressed as (92), where the prices are the optimal prices chosen by the non-signaling firms. In the first period, $\mathrm{Cov}(P_{i1}^{NS}, P_{j1}^{NS}) = \mathrm{Cov}(c_i, P_{j1}^{NS}) = \mathrm{Cov}(c_j, P_{i1}^{NS}) = 0$ for the same reason as in the regime with signaling firms. The social planner's expected first period payoff is (93) with the slope of signaling firms' first-period pricing rule α_1^* being replaced with the slope of non-signaling firms' first-period pricing rule $\alpha_1^{NS} = \frac{1}{2}$. Substituting α_1^{NS} and $\eta_1 = \frac{b}{b^2-e^2}$,

$\eta_2 = \frac{e}{b^2-e^2}$ we obtain

$$\mathbb{E}\left[SW_1^{NS}\right] = \frac{3b}{8}\left(\mathrm{Var}(E(c_i|s_i)) + \mathrm{Var}(E(c_j|s_j))\right) + \Psi. \tag{111}$$

In the second period, since both signaling and non-signaling firms have the same equilibrium pricing functions, the social planner's expected second-period payoff is the same:

$$\mathbb{E}\left[SW_2^{NS}\right] = \mathbb{E}\left[SW_2^{*}\right] = -\frac{\left(12b^2-5e^2\right)be^2}{8\left(4b^2-e^2\right)^2}\left(\mathrm{Var}(E(c_i|s_i)) + \mathrm{Var}(E(c_j|s_j))\right) + \Psi. \tag{112}$$

Adding (111) and (112), the social planner's sum of expected payoffs from the two periods is

$$\sum_{t=1}^{2}\mathbb{E}\left[SW_t^{NS}\right] = \frac{b\left(12b^4-9b^2e^2+2e^4\right)}{2\left(4b^2-e^2\right)^2}\left(\mathrm{Var}(E(c_i|s_i)) + \mathrm{Var}(E(c_j|s_j))\right) + \Psi. \tag{113}$$

Accordingly, the social planner's marginal benefit from Firm i's investment in information is

$$\frac{\partial\sum_{t=1}^{2}\mathbb{E}\left[SW_t^{NS}\right]}{\partial\tau_i} = \frac{b\left(12b^4-9b^2e^2+2e^4\right)}{2\left(4b^2-e^2\right)^2}\frac{1}{\left(\tau_i+\tau_c\right)^2}. \tag{114}$$

Next, we compare social benefit from Firm i's investment with Firm i's individual benefit calculated in (108). The discrepancy between social benefit and individual firm's benefit from investment is

$$\frac{\partial\sum_{t=1}^{2}\mathbb{E}\left[SW_t^{NS}\right]}{\partial\tau_i} - \frac{\partial\mathbb{E}_{s_i}\Pi_i^{NS}(s_i)}{\partial\tau_i} = \frac{b^3}{2\left(4b^2-e^2\right)}\frac{1}{\left(\tau_i+\tau_c\right)^2} > 0. \tag{115}$$

Recall that the discrepancy between the social planner and the signaling firms' marginal benefit from investment is calculated by (104). Taking the difference yields

$$\begin{aligned}
&\left(\frac{\partial\sum_{t=1}^{2}\mathbb{E}\left[SW_t^{NS}\right]}{\partial\tau_i} - \frac{\partial\mathbb{E}_{s_i}\Pi_i^{NS}(s_i)}{\partial\tau_i}\right) - \left(\frac{\partial\sum_{t=1}^{2}\mathbb{E}\left[SW_t^{*}\right]}{\partial\tau_i} - \frac{\partial\mathbb{E}_{s_i}\Pi_i^{*}(s_i)}{\partial\tau_i}\right)\\
&= \left(\frac{b^3}{2\left(4b^2-e^2\right)}\frac{1}{\left(\tau_i+\tau_c\right)^2}\right) - \left(\frac{b\left(16b^4-12b^2e^2+3e^4\right)}{8\left(4b^2-e^2\right)^2}\frac{1}{\left(\tau_i+\tau_c\right)^2}\right)\\
&= \frac{be^2\left(8b^2-3e^2\right)}{8\left(4b^2-e^2\right)^2}\frac{1}{\left(\tau_i+\tau_c\right)^2} > 0.
\end{aligned} \tag{116}$$

Thus, compared to signaling firms, non-signaling firms' investment in information acquisition has a larger externality on consumers and the rivals and hence is less efficient. □

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